Chapter 5 Review

RATES OF CHANGE IN RATIONAL FUNCTION MODELS

CHECK YOUR UNDERSTANDING

1. What is a rational function? Describe in what ways the graphs of rational functions are different from the graphs of polynomial functions.
2. What are the steps for finding a limit of a rational function if substitution fails?
3. How do you determine the equation of **(a)** a vertical asymptote? **(b)** a horizontal asymptote? **(c)** an oblique asymptote?
4. What kinds of discontinuities do rational functions have? When can a discontinuity be removed, and how is this done?
5. State the rule for determining the derivative of a rational function, in symbols and in words. What is this rule called?
6. In what ways can a function fail to have a derivative at a point?
7. Who might need to determine an optimal value? What are the steps for solving an optimization problem using calculus?
8. How can you use the equation of a rational function and its first and second derivatives to sketch the graph of the function by hand?
9. How do you use the graph of a function to sketch the graph of its first derivative? How do you use the graphs of a function's first and second derivatives to sketch the graph of the function?
10. A function is continuous at a point if the function is differentiable at that point. However, a continuous function may not be differentiable at one or more points. Explain, using examples and limit concepts.

ADDITIONAL REVIEW QUESTIONS BY SECTION

5.1 Graphs of Rational Functions

11. Determine the domain and the x- and y-intercepts for each rational function.

(a) $f(x) = \frac{3 - x}{x + 1}$ **(b)** $f(x) = \frac{2x}{3x - 7}$

(c) $f(x) = \frac{4x^2 - 17x - 15}{x}$ **(d)** $f(x) = \frac{2x - 3}{2x^2 + x - 3}$

(e) $f(x) = \frac{3x^2 - 7x + 2}{3x - 1}$ **(f)** $f(x) = \frac{x^2 - 1}{x^3 + 2x^2 - 4x - 8}$

12. Which of the functions in question 11 have

(a) a vertical asymptote?

(b) a horizontal asymptote?

(c) an oblique asymptote?

Give reasons for your answers. Determine the equation of each asymptote. Then sketch the graph for each function.

13. The estimated revenue and cost functions for the manufacture of a new product are $R(x) = -2x^2 + 15x$ and $C(x) = 5x + 8$. Express the *average profit function*, $AP(x) = \frac{P(x)}{x}$, in two different forms. Explain what can be determined from each form. What is the domain of the function in this context? What are the break-even quantities?

5.2 Limits and End Behaviour of Rational Functions

14. Evaluate each limit, where it exists. If a limit does not exist, explain why.

(a) $\lim_{x \to 3} \frac{6 - x^2}{2x}$

(b) $\lim_{x \to 2} \frac{2x}{x^2 - 4}$

(c) $\lim_{x \to -4} \frac{2x^2 + 5x - 12}{x + 4}$

(d) $\lim_{x \to \infty} \frac{1 - 2x^3 - 4x^6}{5x^2 - 2x^6}$

15. Determine the equations of any vertical or horizontal asymptotes for each function. Describe the behaviour of the function on each side of any vertical asymptote.

(a) $f(x) = \frac{x - 5}{2x + 1}$

(b) $g(x) = \frac{x^2 - 4x - 5}{(x + 2)^2}$

(c) $h(x) = \frac{x^2 + 2x - 15}{9 - x^2}$

(d) $m(x) = \frac{2x^2 + x + 1}{x + 4}$

16. Find each limit.

(a) $\lim_{x \to \infty} \frac{3 - 2x}{3x}$

(b) $\lim_{x \to \infty} \frac{x^2 - 2x + 5}{6x^2 + 2x - 1}$

(c) $\lim_{x \to \infty} \frac{7 + 2x^2 - 3x^3}{x^3 - 4x^2 + 3x}$

(d) $\lim_{x \to \infty} \frac{5 - 2x^3}{x^4 - 4x}$

(e) $\lim_{x \to \infty} \frac{2x^5 - 1}{3x^4 - x^2 - 2}$

(f) $\lim_{x \to \infty} \frac{x^2 + 3x - 18}{(x - 3)^2}$

(g) $\lim_{x \to \infty} \frac{x^2 - 4x - 5}{x^2 - 1}$

(h) $\lim_{x \to \infty} \left(5x + 4 - \frac{7}{x + 3}\right)$

5.3 Continuity of Rational Functions

17. Find any points of discontinuity for each function. State which conditions of continuity are not satisfied.

(a) $f(x) = \frac{x}{(x - 5)^2}$

(b) $f(x) = \frac{5}{x^2 + 9}$

(c) $f(x) = \frac{x - 2}{x^2 - 8x + 12}$

(d) $f(x) = \begin{cases} x^3 - 3x^2 + 2x + 4 & \text{if } x \le 0 \\ \frac{x^2 - x + 12}{x + 3} & \text{if } x > 0 \end{cases}$

18. Determine whether or not each function is continuous at $x = -2$.

(a) $f(x) = \begin{cases} \dfrac{1}{x+2} & \text{if } x \neq -2 \\ 0 & \text{if } x = -2 \end{cases}$ (b) $f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x+2} & \text{if } x \neq -2 \\ -5 & \text{if } x = -2 \end{cases}$

19. Define each function so that it is continuous for all real numbers.

(a) $f(x) = \dfrac{x^2 - 4}{x + 2}$ (b) $f(x) = \dfrac{x^2 + x - 12}{x - 3}$

5.4 Rate of Change of a Rational Function — The Quotient Rule

20. Use the quotient rule to find $f'(x)$ for each function.

(a) $f(x) = \dfrac{x + 4}{x - 1}$ (b) $f(x) = \dfrac{2x^3}{x^2 - 3}$

(c) $f(x) = \dfrac{x^3 - x}{2x^2 + x - 1}$ (d) $f(x) = \dfrac{(x + 1)(x - 2)}{3x(x + 4)}$

(e) $f(x) = \dfrac{x^4}{2 - x}$ (f) $f(x) = \dfrac{ax + b}{cx - d}$

(g) $f(x) = \dfrac{x^2 - 4}{3x + 1}$ (h) $f(x) = \dfrac{2x^5 + 3}{x - 2}$

(i) $f(x) = \dfrac{1 - x^4}{x^3}$ (j) $f(x) = \dfrac{1 - \frac{3}{x}}{x - 5}$

21. The position of an object moving along a straight line at time t is given by s. Find the position, velocity, and acceleration at the specified time.

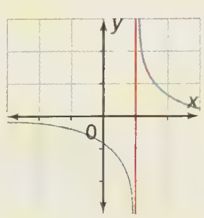
(a) $s(t) = \dfrac{3t}{t + 4}$; $t = 2$ (b) $s(t) = 2t + \dfrac{4}{t + 1}$; $t = 1$

22. The amount of money a family saves is a function, $S(x)$, of its income, x. The *marginal propensity to save* is $S'(x)$. Find the marginal propensity to save if $S(x) = \dfrac{1.5x^2}{5(x + 45\,000)}$.

5.5 Differentiability of Rational and Other Functions

23. Explain why each function is not differentiable at (2, 3).

(a)



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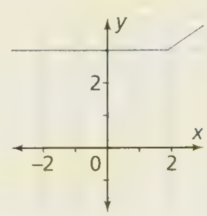

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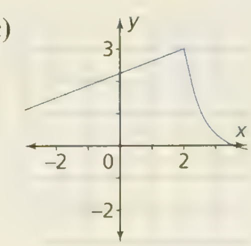


24. Each function is *not* differentiable at $x = 2$. Determine whether the reason is a discontinuity, a corner, a cusp, or a vertical tangent.

(a) $f(x) = \dfrac{x^2 - 4}{x - 2}$ (b) $f(x) = \frac{1}{2}|2 - x| + 1$

(c) $f(x) = \frac{x + 6}{x - 2}$

(d) $f(x) = (x - 2)^{\frac{2}{3}}$

(e) $f(x) = 2\sqrt{4 - 2x}$

(f) $f(x) = (0.5x - 1)^{\frac{1}{3}}$

25. Determine whether each function is differentiable at $x = -1$. Give reasons for your choices.

(a) $f(x) = \frac{7x}{1 - x^2}$

(b) $g(x) = \frac{x + 1}{x^2 - 3x - 4}$

(c) $h(x) = \sqrt[3]{(x - 1)^2}$

(d) $m(x) = |x + 1| - 1$

5.6 Finding Optimal Values for Rational Function Models

26. Craig is designing a rectangular poster that will have an area of 3750 cm^2. The margins at the top and bottom are 12.5 cm wide. The margins at the sides are 7.5 cm wide. What should be the dimensions of the poster if the area of print should be as large as possible?

27. A health drink manufacturer wants to sell its product in 400-mL cans. The metal used for the top and bottom of a can costs \$1.50/m^2. The metal used for the side costs \$0.50/m^2. The metal left over after the circles for the top and bottom of one can are cut out of one rectangle will be scrapped. Find the dimensions of the can that will minimize the cost of materials.

28. A closed rectangular box with a square base will be made as follows. The volume must be 0.9 m^3. The area of the base must not exceed 1.8 m^2. The height of the box must not exceed 0.75 m. Determine the dimensions for **(a)** minimal surface area and **(b)** maximum surface area.

5.7 Sketching Graphs of Rational Functions

29. For $f(x) = \frac{5x}{(x - 1)^2}$, show that $f'(x) = \frac{-5(x + 1)}{(x - 1)^3}$ and $f''(x) = \frac{10(x + 2)}{(x - 1)^4}$. Use the function and its derivatives to determine the domain, intercepts, asymptotes, intervals of increase and decrease, and concavity, and to locate any turning points and points of inflection. Use this information to sketch the graph of f.

30. Determine the first and second derivatives for each function. Then analyze each function and sketch its graph.

(a) $f(x) = \frac{1 - 2x^2}{x^3}$

(b) $f(x) = \frac{x}{(x - 3)^2}$

(c) $f(x) = \frac{x + 2}{x + 3}$

(d) $f(x) = 2x + \frac{1}{x^3}$

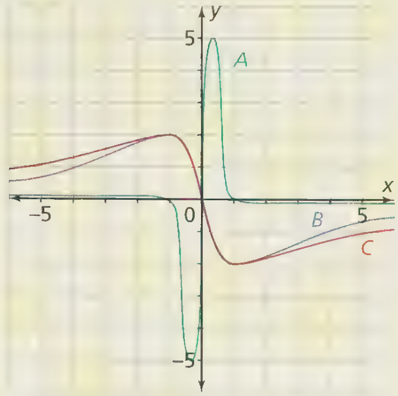
(e) $f(x) = \frac{x}{2 - 3x}$

(f) $f(x) = \frac{x^2 - 1}{x^2 + 2}$

(g) $f(x) = \frac{2x}{x^2 + 1}$

(h) $f(x) = \frac{x^2 - x}{x + 2}$

31. The graphs of a function and its derivatives are shown. Which is which? Explain how you can tell.



REVIEW QUESTIONS BY ACHIEVEMENT CHART CATEGORIES

Knowledge and Understanding

32. Evaluate each limit, if it exists. If the limit does not exist, explain why.

(a) $\lim\limits_{x \to -2} \dfrac{3x}{2 + x^2}$

(b) $\lim\limits_{x \to 1} \dfrac{x + 1}{x^2 - 1}$

(c) $\lim\limits_{x \to 2} \dfrac{x^2 + 4x - 12}{x^2 - 4}$

(d) $\lim\limits_{x \to \infty} \dfrac{2x^5 - 3x^4 + 5x}{3x^3 - 7x^5}$

33. Find the equation of the tangent line to the graph of $f(x) = \dfrac{8x^2 - 3}{x^2 + 4}$ at the point where $x = -1$.

34. For $f(x) = \dfrac{x^2 + 3}{x - 1}$,

(a) show that $f'(x) = \dfrac{x^2 - 2x - 3}{(x - 1)^2}$ and $f''(x) = \dfrac{8}{(x - 1)^3}$

(b) use the information given by $f(x)$ and its derivatives to graph $f(x)$ without graphing technology

Communication

35. Let $f(x) = \dfrac{-x - 3}{x^2 - 5x - 14}$, $g(x) = \dfrac{x - x^3}{x - 3}$, $h(x) = \dfrac{x^3 - 1}{x^2 + 4}$, and $r(x) = \dfrac{x^2 + x - 6}{x^2 - 16}$. How can you can tell from its equation which of these functions has

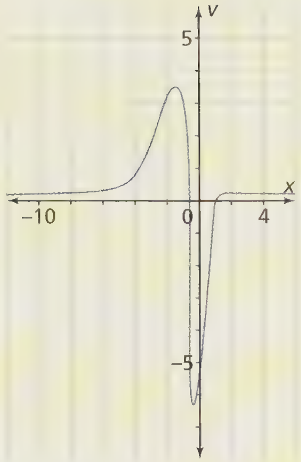
(a) a horizontal asymptote?

(b) an oblique asymptote?

(c) no vertical asymptote?

Explain. Determine the equations of all asymptote(s) for each function. Describe the behaviour of each function close to its asymptotes.

36. The function $c(t) = \frac{7t}{2t^2 + 1}$ represents the concentration, $c(t)$, of a drug in the bloodstream t hours after it is taken orally. Determine $c'(t)$ and $c''(t)$. Explain, using numerical examples, the information these two derivative functions provide.



37. The graph of $y = f'(x)$ is shown on the left. Sketch possible graphs for $y = f(x)$ and $y = f''(x)$. Explain how you sketched the graphs.

Application

38. For what values of A does $f(x) = \frac{x^2 + 2x + A}{x^2 - 4x - 5}$ have a removable discontinuity? For each value of A, redefine the function to make it continuous.

39. The concentration of a pollutant in polluted water is 2 g/L. The polluted water flows into a large tank that initially holds 800 L of pure water. Ten litres of polluted water per minute flow into the tank. Determine the concentration of pollutant in the tank after t minutes. Describe how this concentration is changing after 3 hours; after 12 hours. What assumptions do you need to make?

40. A rectangular wooden bedding chest will be built so that its length is 2.5 times its width. The top, front, and two sides of the chest will be oak. The back and bottom of the chest will be aromatic cedar. The volume of the chest must be 0.5 m^3. Oak costs 1.5 times as much as cedar. Find the dimensions that will minimize the cost of the chest.

Thinking, Inquiry, Problem Solving

41. Suppose $p(x)$ and $q(x)$ are rational functions and $\lim_{x \to +\infty} [p(x)] = \infty$ and $\lim_{x \to \infty} [q(x)] = \infty$. Is it always true that $\lim_{x \to \infty} [p(x) - q(x)] = 0$? Investigate using different functions for $p(x)$ and $q(x)$.

42. Rolle's theorem states: Let f be differentiable on $a < x < b$ and continuous on $a \leq x \leq b$. If $f(b)$ and $f(a)$ are both 0, there is at least one number c in $a < x < b$ for which $f'(c) = 0$. Does $f(x) = \frac{x^2 - 9x + 8}{x}$ satisfy Rolle's theorem on any interval? If so, what are the values of a, b, and c?

43. For function $f(x)$, $f''(x)$ is continuous and positive for all values of x, and $f(x)$ is negative for all values of x. For $g(x) = \frac{1}{f(x)}$, show that $g(x)$ is always concave down.

Chapter 5 Review Test

RATES OF CHANGE IN RATIONAL FUNCTION MODELS

1. Let $p(x) = \dfrac{3x^3 - 5}{4x^2 + 1}$, $q(x) = \dfrac{3x - 1}{x^2 - 2x - 3}$, $r(x) = \dfrac{x^2 - 2x - 8}{x^2 - 1}$, and $s(x) = \dfrac{x^3 + 2x}{x - 2}$.
   (a) For each function, determine its asymptotes and identify their type (vertical, horizontal, or oblique).
   (b) Graph $y = r(x)$, showing clearly the asymptotes and the intercepts.

2. **Knowledge and Understanding:** Evaluate each limit, if it exists. If it does not exist, explain why.
   (a) $\lim\limits_{x \to 0} \dfrac{4}{x + 2}$
   (b) $\lim\limits_{x \to 4} \dfrac{x}{16 - x^2}$
   (c) $\lim\limits_{x \to 1} \dfrac{x^5 - 1}{x^2 - 1}$
   (d) $\lim\limits_{x \to \infty} \dfrac{1 - 3x^3 - 2x^5}{5x^5 - 8x^4}$

3. Describe the discontinuities, if any, for each function.
   (a) $f(x) = |x + 3|$
   (b) $f(x) = \dfrac{x + 2}{x^2 - 5x - 14}$

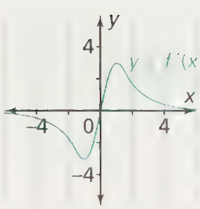
4. **Application:** The population of Newton is modelled by $P(t) = \dfrac{20(4t + 3)}{2t + 5}$, where $P(t)$, the population, is measured in thousands and t is the time in years since 1990.
   (a) Is the population increasing or decreasing? Justify your answer.
   (b) Determine the rate of change of the population in 1996 to three decimal places.
   (c) The town will need its own transit system if the population exceeds 50 000. Will Newton's population ever exceed 50 000? Explain.

5. **Thinking, Inquiry, Problem Solving:** Determine the equations of the two tangents to the curve $y = \dfrac{x}{(1 - x)^2}$ that pass through (1, 1).

6. At what values of x is each function *not* differentiable? Explain.
   (a) $f(x) = \dfrac{7}{2x - 1}$
   (b) $f(x) = \dfrac{x^2 - 5x - 6}{x^2 - 1}$

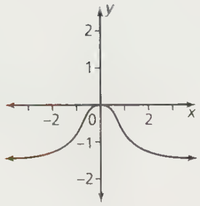
7. A cylindrical vat must hold 5 m^3 of liquid cake mix. The vat must be wider than it is tall, but no more than 3 m in diameter. What dimensions will use the least amount of material?

8. For $f(x) = \dfrac{x^3 + 8}{x}$, determine the domain, intercepts, asymptotes, intervals of increase and decrease, and concavity. Locate any critical points and points of inflection. Use this information to sketch the graph of $f(x)$.



9. **Communication:** Explain how you can use this graph of $y = f'(x)$ to sketch a possible graph of the original function $y = f(x)$.

19. (a)



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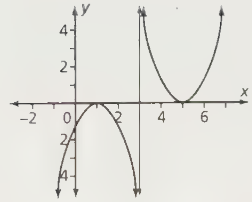

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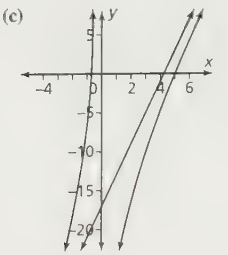

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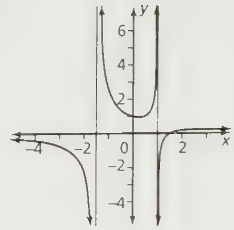

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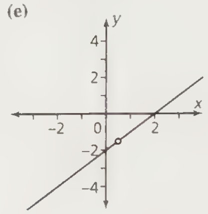

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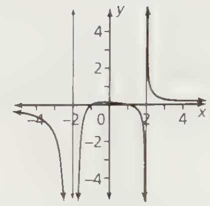


20. (a) A: f, C: f', B: f''; when derivative is taken, degree of denominator increases by 1

(b) F: f, E: f', D: f''; when derivative is taken, degree of denominator increases by 1

21. (a) true; if $f'(3) = 0$, graph changes from pos. to neg. or neg. to pos. in which case a local max. or local min. exists at $x = 3$

(b) false; graph could have oblique asymptote if highest degree of x in numerator is exactly one greater than highest degree of x in denominator

(c) true; if $f'(2) = 0$, concavity at $x = 2$ changes from concave up to concave down or concave down to concave up. This makes $x = 2$ a point of inflection.

22. Analysis will vary slightly.

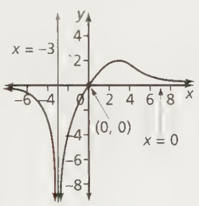
23. $a = 1$; $b = 2$

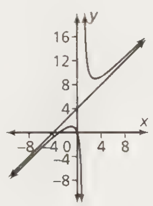
24. disagree; counterexample is question 20

25. $f(x) = \frac{x-1}{x}$

26. $f(x) = \frac{4}{1+x^2}$

27. (a)





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11. (a) $D = \{x \mid x \neq -1, x \in \mathbf{R}\}$; x-int. and y-int.: 3

(b) $D = \{x \mid x \neq \frac{7}{3}, x \in \mathbf{R}\}$; x-int. and y-int.: 0

(c) $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$; x-int.: 5 and $-\frac{3}{4}$

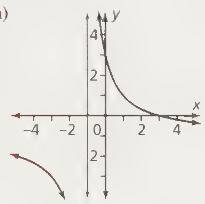
(d) $D = \{x \mid x \neq -\frac{3}{2}, 1, x \in \mathbf{R}\}$; x-int.: $\frac{3}{2}$; y-int.: 1

(e) $D = \{x \mid x \neq \frac{1}{3}, x \in \mathbf{R}\}$; x-int.: 2; y-int.: -2

(f) $D = \{x \mid x \neq -2, 2, x \in \mathbf{R}\}$; x-int.: -1 and 1; y-int.: $\frac{1}{8}$

12. (a) (a), (b), (d), (f) (b) (a), (b), (d), (f) (c) (c), (e)

(a)



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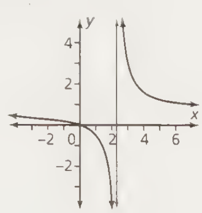


13. $AP(x) = \frac{-2(x-1)(x-4)}{x}$: easy to see what break-even quantities are; $AP(x) = -2x + 10 - \frac{8}{x}$: easy to tell what curve will look like in long run.; curve will act as line $AP(x) = -2x + 10$ as it approaches infinity; break-even points: $x = 1$, $x = 4$

14. (a) $-\frac{1}{2}$ (b) does not exist; vert. asymp. at $x = 2$

(c) -11 (d) 2

15. (a) vert. asymp.: $x = -\frac{1}{2}$; $x \to -\frac{1}{2}^-$, $f(x) \to \infty$; $x \to -\frac{1}{2}^+$, $f(x) \to -\infty$; hor. asymp.: $y = \frac{1}{2}$

(b) vert. asymp.: $x = -2$; $x \to -2^-$, $g(x) \to \infty$; $x \to -2^+$, $g(x) \to \infty$; hor. asymp.: $y = 1$

(c) vert. asymp.: $x = -3$; $x \to -3^-$, $h(x) \to \infty$; $x \to -3^+$, $h(x) \to -\infty$; hor. asymp.: $y = -1$

(d) vert. asymp.: $x = -4$; $x \to -4^-$, $m(x) \to -\infty$; $x \to -4^+$, $g(x) \to \infty$; hor. asymp.: none

16. (a) $-\frac{2}{3}$ (b) $\frac{1}{6}$ (c) -3 (d) 0

(e) ∞ (f) 1 (g) 1 (h) ∞

17. (a) $x = 5$, conditions 1 & 2 (b) none

(c) $x = 6$, conditions 1 & 2, $x = 2$, condition 2

(d) none

18. (a) discontinuous (b) continuous

19. (a) $\begin{cases} \frac{x^2-4}{x+2}, x \neq -2 \\ -4, x = -2 \end{cases}$ (b) $\begin{cases} \frac{x^2+x-12}{x-3}, x \neq 3 \\ 7, x = 3 \end{cases}$

20. (a) $f'(x) = -\frac{5}{(x-1)^2}$ (b) $f'(x) = \frac{2x^2(x-3)(x+3)}{(x^2-3)^2}$

(c) $f'(x) = \frac{2x^4 + 2x^3 - x^2 + 1}{(2x^2 + x - 1)^2}$ (d) $f'(x) = \frac{5x^2 + 4x + 8}{3(x^2 + 4x)^2}$

(e) $f'(x) = \frac{x^3(8 - 3x)}{[illegible]}$ (f) $f'(x) = -\frac{ad + bc}{[illegible]}$

(g) $f'(x) = \frac{3x^2 + 2x + 12}{(3x+1)^2}$ (h) $f'(x) = \frac{8x^5 - 20x^4 - 3}{(x-2)^2}$

(i) $f'(x) = \frac{-x^4 - 3}{x^4}$ (j) $f'(x) = \frac{-x^2 + 6x - 15}{x^2(x-5)^2}$

21. (a) $s(2) = 1$, $v(2) = \frac{1}{3}$, $a(2) = -\frac{1}{9}$

(b) $s(1) = 4$, $v(1) = 1$, $a(1) = 1$

22. $S(x) = \frac{0.3x^2 + 27\,000}{(x + 45\,000)^2}$

23. (a) vert. asymp. at $x = 2$; not defined there

(b) corner at $x = 2$, not defined there

(c) cusp at $x = 2$

24. (a) discontinuity, hole (b) corner

(c) discontinuity, vertical asymptote (d) cusp

(e) discontinuity, restricted domain (f) vertical tangent

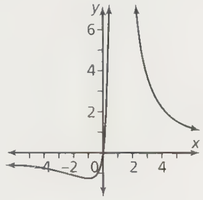
25. (a) not differentiable, not defined
(b) not differentiable, not defined
(c) differentiable (d) not differentiable, corner

26. 47.4 cm × 79.1 cm

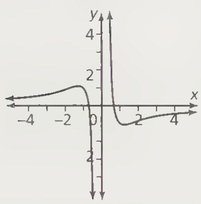
27. radius 2.55 cm; height 19.58 cm

28. (a) $\sqrt{1.2}$ m × $\sqrt{1.2}$ m × 0.75 m; 5.69 m^3
(b) $\sqrt{1.8}$ m × $\sqrt{1.8}$ m × 0.5 m; 6.28 m^3

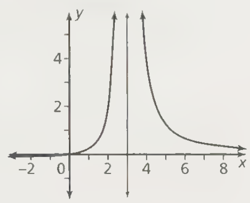
29. D = $\{x \mid x \neq 1, c \in \mathbf{R}\}$; x-int. and y-int.: 0; vert. asymp.: $x = 1$; $x \to 1^-, f(x) \to \infty$; $x \to 1^+, f(x) \to \infty$; hor. asymp.: $y = 0$; decreasing: $x < -1, x > 1$; increasing: $-1 < x < 1$; local min: $(-1, -1.25)$; concave down: $x < -2$; concave up: $-2 < x < 1, x > 1$; point of inflection: $(-2, -1.11)$



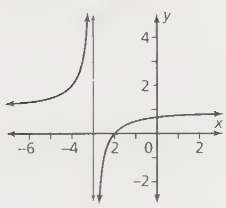
30. (a) $f'(x) = \frac{2x^2 \quad 3}{x^4}$; $f''(x) = \frac{-4(x^2 \quad 3)}{x^5}$



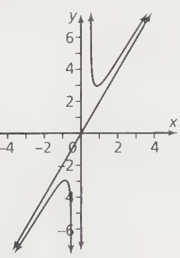
(b) $f'(x) = \frac{(x + 3)}{(x - 3)^3}$; $f''(x) = \frac{2(x + 6)}{(x - 3)^4}$



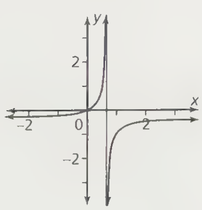
(c) $f'(x) = \frac{1}{(x + 3)^2}$; $f''(x) = -\frac{2}{(x + 3)^3}$



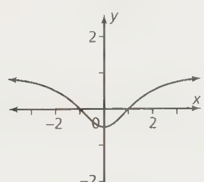
(d) $f'(x) = \frac{2x^4 - 3}{x^4}$; $f''(x) = \frac{12}{x^5}$



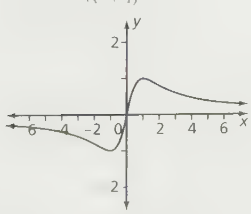
(e) $f'(x) = \frac{2}{(2 \quad 3x)^2}$; $f''(x) = \frac{12}{(2 \quad 3x)^3}$



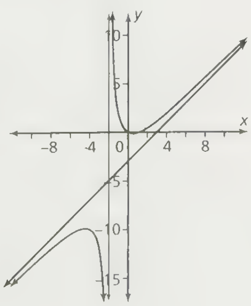
(f) $f'(x) = \frac{6x}{(x^2 + 2)^2}$; $f''(x) = \frac{-6(3x^2 - 2)}{(x^2 + 2)^3}$



(g) $f'(x) = \frac{2(1 - x^2)}{(x^2 + 1)^2}$; $f''(x) = \frac{4x(x^2 - 3)}{(x^2 + 1)^3}$



(h) $f'(x) = \frac{x^2 + 4x - 2}{(x + 2)^2}$; $f''(x) = \frac{12}{(x + 2)^3}$

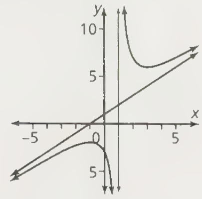


31. C: f, B: f'; A: f''; as derivative is taken, degree of denominator increases by one

32. (a) −1 (b) limit does not exist
(c) 2 (d) $-\frac{2}{7}$

33. $y = -\frac{14}{5}x \quad \frac{9}{5}$

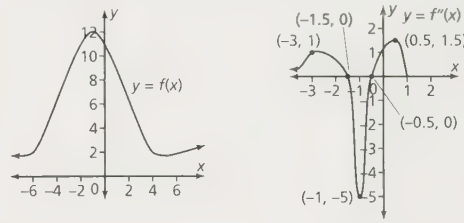
34. (b)



35. (a) $f(x)$, $r(x)$ (b) $h(x)$ (c) $h(x)$
$f(x)$: vert. asymp.: $x = 7$, $x = -2$; $x \to -2^-, f(x) \to -\infty$; $x \to -2^+, f(x) \to \infty$; $x \to 7^-, f(x) \to \infty$; $x \to 7^+, f(x) \to -\infty$; hor. asymp.: $y = 0$
$g(x)$: vert. asymp.: $x = 3$; $x \to 3^-$, $g(x) \to -\infty$; $x \to 3^+$, $g(x) \to -\infty$; $h(x)$: oblique asymp.: $y = x$
$r(x)$: vert. asymp.: $x = -4$, $x = 4$; $x \to -4^-$, $r(x) \to \infty$; $x \to -4^+$, $r(x) \to -\infty$; $x \to 4^-$, $rf(x) \to -\infty$; $x \to 4^+$, $rf(x) \to \infty$; $r(x)$ has a hor. asymp. at $x = 1$

36. $c'(t) = \frac{7(1 - 2t^2)}{(2t^2 + 1)^2}$; $c''(t) = \frac{28(2t^3 - 3)}{(2t^2 + 1)^3}$; $c'(t)$: rate of change of concentration of drug in bloodstream t h after taken orally; $c''(t)$: how rate of change of concentration of drug in bloodstream is changing

37. Explanations will vary.



38. $A = -35$: $f(x) = \begin{cases} \frac{x^2 + 2x - 35}{x^2 - 4x - 5}, x \neq 5 \\ 2, x = 5 \end{cases}$;
$A = 1$: $f(x) = \begin{cases} \frac{x^2 + 2x + 1}{x^2 - 4x - 5}, x \neq -1 \\ 0, x = -1 \end{cases}$

39. 1.39 g/L/min, 1.80 g/L/min; assume tank has infinite capacity

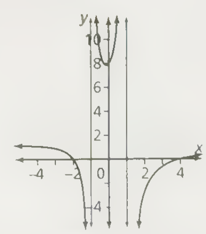
40. 0.529 m × 0.715 m × 1.323 m

41. no; Examples will vary.

42. yes; $a = 1$, $b = 8$, $c = 2\sqrt{2}$ or 2.8

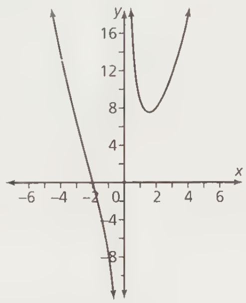
Chapter 5 Review Test, page 426

1. (a) p has oblique asymp.: $y = 0.75x$; q has hor. asymp.: $y = 0$, and vert. asymp., $x = -1$ and $x = 3$ r has a hor. asymp., $y = 1$, and vert. asymp., $x = \pm 1$; s has vert. asymp. at $x = 2$
(b)

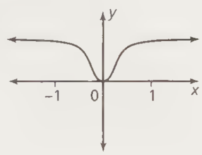


2. (a) 2 (b) does not exist, vertical asymptote at $x = 4$
(c) $\frac{5}{\ }$ (d) $\frac{2}{\ }$

3. (a) none (b) discontinuous at $x = 7$, vertical asymptote
4. (a) increasing, $P'(t) \geq 0$ (b) 969 people per year
(c) no; $P(t) = 50$ has negative solution
5. $y = x$; $y = -\frac{5}{27}x + \frac{32}{27}$
6. (a) $x = \frac{1}{2}$: vert. asymp.
(b) $x = -1$: not defined; $x = 1$: vert. asymp.
7. radius 1.5 m, height 0.707 m
8. $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$; x-int.: -2; y-int.: 8; vert. asymp.: $x = 0$; $x \to 0^-, f(x) \to -\infty$; $x \to 0^+$, $f(x) \to \infty$; hor. asymp.: none; increasing: $x > 1.59$; decreasing: $x < 0$, $0 < x < 1.59$, local min.: (1.59, 7.56); concave up: $x < -2$, $x > 0$; concave down: $-2 < x < 0$; inflection point: $(-2, 0)$

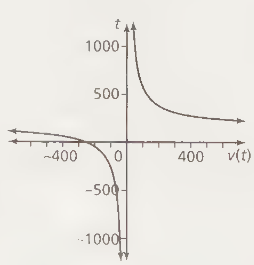


9. if $f'(x) > 0$, then $f(x)$ is increasing; if $f'(x) < 0$, then $f(x)$ is decreasing; for a stationary point, $f'(x) = 0$; changes from neg. to pos. are local min. points; slope gives concavity



Cumulative Review 2, page 427

1. (a) $y' = 12x$; $x = 0$; increasing $x > 0$, decreasing $x < 0$
(b) $y' = -10x + 20$; $x = 2$; increasing $x < 2$, decreasing $x > 2$
(c) $y' = 12x + 16$; $x = -\frac{4}{3}$; increasing $x > -\frac{4}{3}$; decreasing $x < -\frac{4}{3}$
(d) $y' = 6x^2 - 24$; $x = \pm 2$; increasing $x < -2$, $x > 2$; decreasing $-2 < x < 2$
(e) $y' = -8x^3 + 8x$; $x = 0, \pm 1$; increasing $x < -1$, $0 < x < 1$; decreasing $-1 < x < 0$, $x > 1$
(f) $y' = 2x^3 + x^2 - 13x + 6$; $x = \frac{1}{2}, 2, -3$; increasing $-3 < x < \frac{1}{2}$, $x > 2$; decreasing $x < -3$, $\frac{1}{2} < x < 2$
2. $y = -3x + 1$
3. (a) 21 (b) 18 (c) -16
(d) -217 (e) -136 (f) 28 144
4. 12.6; rate of change of the height of the arrow with respect to time at 3 s; 2.8 m/s
5. 19.6 m/s; 19.6 m/s; 53.7 m/s
6. \$26 billion/year
7. 10 m; 13 m/s, 28 m/s; never; advancing: $t \geq 0$; retreating: never; 218.3
8. decelerating; velocity is pos. and acceleration is neg.
9. 17.54 cm
10. \$40, yes; 100 empty seats
11. \$200 000
12. (a) $f'(x) = \frac{2}{(1 - x)^2}$ (b) $f'(x) = \frac{4}{(x + 3)^2}$ (c) $f'(x) = \frac{2x^{[illegible]} + 1}{3x^2}$
13. $D = \{t \mid t \neq -2, t \in \mathbf{R}\}$; int.: $(-254.8, 0)$, $(0, 21\ 150)$; vert. asymp.: $t = -2$; hor. asymp.: $V(t) = 166$; worth \$166 in the long run



14. (a) $m(x) = \frac{1}{3x}$, $y = 0$ (b) $m(x) = 3$, $y = 3$
15. Answers will vary. Example: $f(x) = \begin{cases} \frac{x^2 - 3x - 4}{x - 4}, & x > -1 \\ x - 4, & x \leq 1 \end{cases}$
16. $f(x) = \begin{cases} \frac{1}{x}, & \text{if } x < 1 \\ x, & \text{if } x \geq 1 \end{cases}$: left- and right-sided limits of derivative are not same;
$f(x) = \begin{cases} \frac{1}{x}, & \text{if } x < 1 \\ 2 - x, & \text{if } x \geq 1 \end{cases}$: left- and right-sided limits of derivative are same
17. 64 chairs/week; 50 chairs/week
18. $x \doteq 2.4495$; $R(x)$ has point of inflection here.

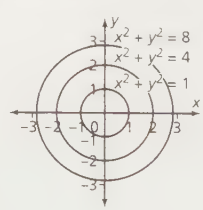
Chapter 6

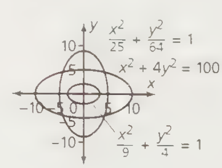
Getting Ready, page 432

1. (a) $m = 2.25$ (b) $m = 3$ (c) $m = 2$
Line (b); magnitude of slope is greatest
2. (a) $f^{-1}(x) = \{(-9, 0), (-7, 2), (3, 1), (4, -1), (5, -2)\}$;
D of f: $x \in \{-2, -1, 0, 1, 2\}$; R of f: $y \in \{-9, -7, 3, 4, 5\}$;
D of f^{-1}: $x \in \{-9, -7, 3, 4, 5\}$; R of f^{-1}: $y \in \{-2, -1, 0, 1, 2\}$;
D of f = R of f^{-1}; R of f = D of f^{-1}
(b)

x	10	5	2	1	2
$g^{-1}(x)$	−3	−2	−1	0	1

D of f: $x \in \{-3, -2, -1, 0, 1\}$; R of f: $y \in \{1, 2, 5, 10\}$;
D of f^{-1}: $x \in \{1, 2, 5, 10\}$; R of f^{-1}: $y \in \{-3, -2, -1, 0, 1\}$;
D of f = R of f^{-1}; R of f = D of f^{-1}
(c) D of f: $x \in \{1, 3, 4, 7\}$; R of f: $y \in \{0, 2, 5\}$;
D of f^{-1}: $x \in \{0, 2, 5\}$; R of f^{-1}: $y \in \{1, 3, 4, 7\}$;
D of f = R of f^{-1}; R of f = D of f^{-1}
3. (a) $D = x \in \mathbf{R}$; $R = y \in \mathbf{R}$ (b) $D = x \in \mathbf{R}$; $R = \{y \mid y \geq 2, y \in \mathbf{R}\}$
(c) $D = \{x \mid x \geq 0, x \in \mathbf{R}\}$; $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$
(d) $D = x \in \mathbf{R}$; $R = \{y \mid y \geq -68.918, y \in \mathbf{R}\}$
(e) $D = s \in \mathbf{R}$; $R = \{y \mid y \geq 0.147, y \in \mathbf{R}\}$
(f) $D = \{x \mid x \neq 0.906, x \in \mathbf{R}\}$; $R = y \in \mathbf{R}$
(g) $D = \{t \mid t \neq 1, t \in \mathbf{R}\}$; $R = \{y \mid y \geq -0.406, y \in \mathbf{R}\}$
(h) $D = \{u \mid u \geq 2 \text{ or } u < -6, u \in \mathbf{R}\}$; $R = \{y \mid y \geq 0, y \neq 1, y \in \mathbf{R}\}$
(i) $D = a \in \mathbf{R}$; $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$
4. (a) $f^{-1}(x) = \frac{x + 3}{2}$ (b) $g^{-1}(x) = \frac{3 \pm \sqrt{41 - 16x}}{8}$
(c) $h^{-1}(x) = \frac{x^2 - 9}{9}$, $x \geq 0$ (d) $k^{-1}(x) = 4x - 1$
(e) $q^{-1}(x) = (-1 - x)^3$ (f) $r^{-1}(x) = \pm\sqrt{x^2 + 16}$, $x \geq 0$
5. (a) they all have the same centre, (0, 0), but with radii 2, $2\sqrt{2}$, and 1 respectively
(b) they all have the same centre, (0, 0), but with different minor and major axes





6. (a) $f'(x) = 0$ (b) $f^{-1}(x) = \frac{1}{[illegible]}$
(c) $f'(x) = 4$ (d) $f'(x) = 23x^{22}$