4.1 Analyzing a Polynomial Function: Intervals of Increase and Decrease

SETTING THE STAGE

Explore the concepts in this lesson in more detail using Exploration 6 on page 570.

The terms *increasing* and *decreasing* describe how a function changes over an interval. For example, the temperature usually increases from May to July and decreases from October to December.

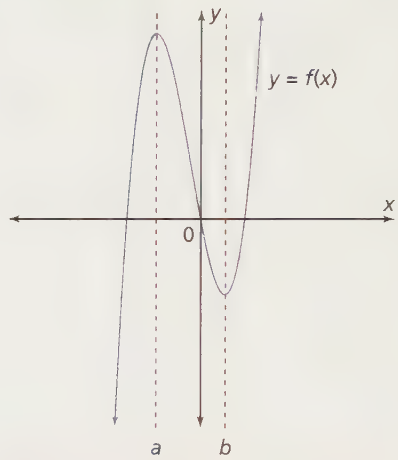
The derivative of a function at a specific point is the slope of the tangent line at this point. The slopes, where the function is increasing and where it is decreasing, are different.

In this section, you will use the first derivative to algebraically determine where a function is increasing or decreasing.

EXAMINING THE CONCEPT

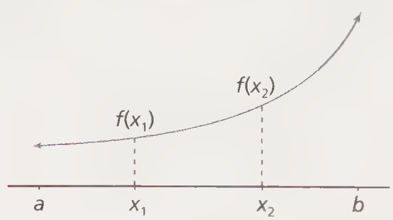
Increasing and Decreasing Functions

The values of $y = f(x)$ increase on the open intervals $-\infty < x < a$ and $b < x < \infty$. In these two intervals, the graph of $y = f(x)$ rises from left to right. The function decreases on the open interval $a < x < b$. In this interval, the graph falls from left to right.



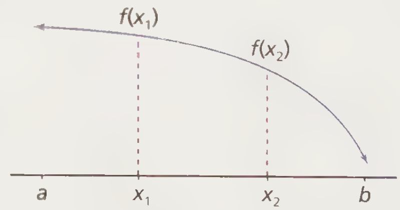
Definition of Increasing and Decreasing Functions

A function $f(x)$ is **increasing** on the interval I $(a < x < b)$, if $f(x_1) < f(x_2)$ for all pairs of numbers x_1 and x_2 in I such that $x_1 < x_2$.



Function f increases on an interval if the values of $f(x)$ increase as x increases.

A function $f(x)$ is **decreasing** on the interval I $(a < x < b)$, if $f(x_1) > f(x_2)$ for all pairs of numbers x_1 and x_2 in I such that $x_1 < x_2$.

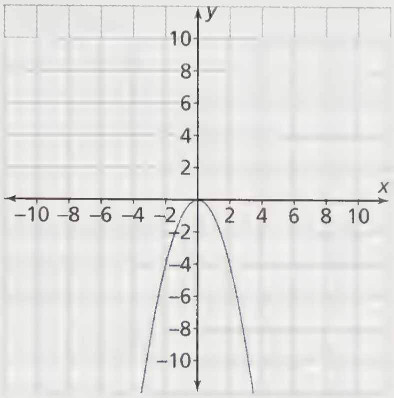


Function f decreases on an interval if the values of $f(x)$ decrease as x increases.

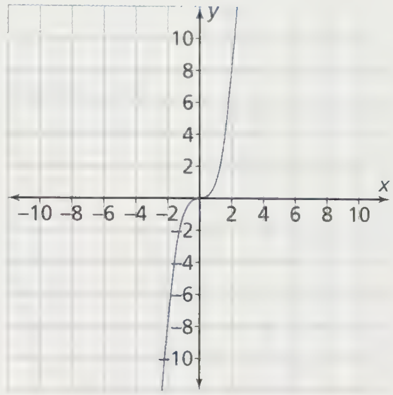
Example 1 **Determining Intervals of Increase and Decrease Graphically**

State the intervals of increase and decrease.

(a)



(b)



Solution

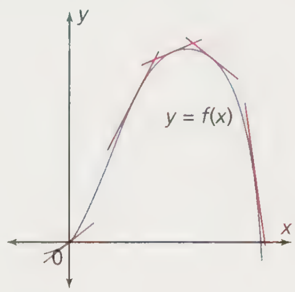
(a) The graph is rising, and thus increasing, on the interval $-\infty < x < 0$. The graph is falling, and thus decreasing, on the interval $0 < x < \infty$. The graph changes from increasing to decreasing at $x = 0$.

(b) For all values of x, the graph is always rising. Therefore, the function is increasing on the interval $-\infty < x < \infty$.

EXAMINING THE CONCEPT

Using the Derivative to Determine Intervals of Increase or Decrease

The derivative of a function at a point is the slope of the tangent line at that point. How does the derivative change at various points? Study this diagram. Where is the slope of each tangent line positive? negative?



Where $f(x)$ is increasing and $f'(x) > 0$, the slope of each tangent line is *positive*. These lines slope up to the right. Where $f(x)$ is decreasing and $f'(x) < 0$, the slope of each tangent line is *negative*. These lines slope down to the right.

This pattern suggests that you can use the sign of the derivative, $f'(x)$, to determine where a function is increasing or decreasing.

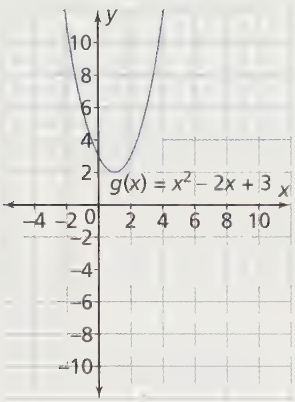
> **Test for Increasing and Decreasing Functions**
>
> If $f'(x) > 0$ for all x in that interval, then f is *increasing* on the interval $a < x < b$.
> If $f'(x) < 0$ for all x in that interval, then f is *decreasing* on the interval $a < x < b$.

Example 2 Using the Derivative to Determine Intervals of Increase or Decrease

Determine the intervals where each function increases and decreases.

(a) $g(x) = x^2 - 2x + 3$ **(b)** $y = -\frac{2}{3}x^3 + x^2 + 12x - 1$

Solution



(a) Start by finding the derivative.

$$g'(x) = \frac{d}{dx}(x^2 - 2x + 3)$$
$$= 2x - 2$$

The function $g(x)$ increases when $g'(x) > 0$ and decreases when $g'(x) < 0$.

i. increasing when $2x - 2 > 0$, $2x > 2$, $x > 1$

ii. decreasing when $2x - 2 < 0$, $2x < 2$, $x < 1$

The function $g(x)$ is quadratic. The graph of the function is a parabola that opens up. The y-intercept is 3, since $g(0) = 3$. The graph changes direction when $x = 1$ at the vertex $(1, 2)$.

The function $g(x)$ decreases when $x < 1$ and increases when $x > 1$.

(b) The function $y = -\frac{2}{3}x^3 + x^2 + 12x - 1$ increases when $y' > 0$ and decreases when $y' < 0$.

$$\begin{aligned} y' &= \frac{d}{dx}\left(-\frac{2}{3}x^3 + x^2 + 12x - 1\right) \\ &= 3\left(-\frac{2}{3}\right)x^2 + 2x + 12 \\ &= -2x^2 + 2x + 12 \qquad \text{Factor.} \\ &= -2(x^2 - x - 6) \\ &= -2(x - 3)(x + 2) \end{aligned}$$

The function is increasing when $-2(x - 3)(x + 2) > 0$.

$-2(x - 3)(x + 2) = 0$ when $x = 3$ or when $x = -2$.

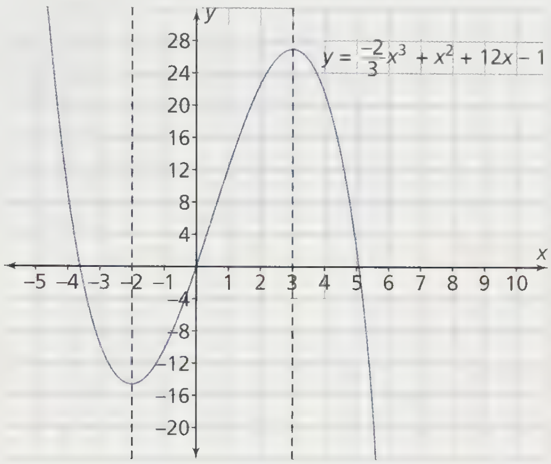
The numbers 3 and -2 divide the x-axis into three intervals: $x < -2$, $-2 < x < 3$, and $x > 3$.

What is the sign of the derivative for each interval? Choose a value for x in each interval. Substitute this value into each factor and solve. Note the sign. Then multiply the signs to get the sign of the derivative.

	Intervals		
	$x < -2$	$-2 < x < 3$	$x > 3$
-2	−	−	−
$(x - 3)$	−	−	+
$(x + 2)$	−	+	+
Sign of y'	(−)(−)(−) = −	(−)(−)(+) = +	(−)(+)(+) = −
Behaviour of y	decreasing ↘	increasing ↗	decreasing ↘

Notice that the intervals are in order from left to right along the x-axis, so you can see where the function increases or decreases from left to right. This helps when graphing the function.

A graph confirms the analysis.



The function $y = -\frac{2}{3}x^3 + x^2 + 12x - 1$ is

- decreasing on $x < -2$
- increasing on $-2 < x < 3$
- decreasing on $x > 3$

Example 3 Using the First Derivative to Analyze a Model

ROXS, a music store, predicts that every dollar increase in the price of any CD will cause sales to decrease by 10 000 units a year. The store now sells 300 000 CDs a year at \$15 each.

(a) Develop a model that represents the music store's sales revenue.

(b) Using this model, determine when revenue will increase and when it will decrease.

(c) The store's manager is thinking about raising the price of any CD by \$2. At this new price, what is the rate of change in revenue?

Solution

(a) revenue = price × units sold

Let x represent the price increase in dollars. Then price $= (15 + x)$ and units sold $= (300\,000 - 10\,000x)$.

$$\begin{aligned} \text{revenue} &= (15 + x)(300\,000 - 10\,000x) \\ &= 4\,500\,000 - 150\,000x + 300\,000x - 10\,000x^2 \\ &= 4\,500\,000 + 150\,000x - 10\,000x^2 \end{aligned}$$

Let R represent revenue. The model for sales revenue is

$$R = 4\,500\,000 + 150\,000x - 10\,000x^2$$

(b) Revenue will increase when $R' > 0$. Revenue will decrease when $R' < 0$.

$$\begin{aligned} R' &= \frac{d}{dx}(4\,500\,000 + 150\,000x - 10\,000x^2) \\ &= 150\,000 - 20\,000x \end{aligned}$$

R is increasing when

$$\begin{aligned} 150\,000 - 20\,000x &> 0 \\ \frac{-20\,000x}{-20\,000} &> \frac{-150\,000}{-20\,000} \\ x &< 7.5 \end{aligned}$$

R is decreasing when

$$\begin{aligned} 150\,000 - 20\,000x &< 0 \\ \frac{-20\,000x}{-20\,000} &< \frac{-150\,000}{-20\,000} \\ x &> 7.5 \end{aligned}$$

Revenue will increase if the price is raised by any amount up to \$7.50. Revenue will decrease if the price is raised by more than \$7.50.

(c) The derivative represents the (instantaneous) rate of change at a point. To find the rate of change for a price increase of \$2, evaluate $R'(2)$.

$$\begin{aligned} R'(2) &= 150\,000 - 20000(2) \\ &= 150\,000 - 40\,000 \\ &= 110\,000 \end{aligned}$$

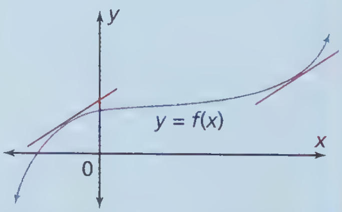
Revenue would increase at a rate of \$110 000 per dollar increase in price.

CHECK, CONSOLIDATE, COMMUNICATE

1. How can you estimate, from the graph of a polynomial function, the intervals where the function is increasing? decreasing?
2. What is always true about the slopes of all tangent lines on a section of a curve that is rising? falling?
3. Graph the function that is increasing on the interval $-2 < x < 2$, decreasing on the interval $2 < x < 4$, and increasing on the interval $4 < x < 7$. Draw a smooth curve. Suggest the degree of a polynomial function that fits this description.

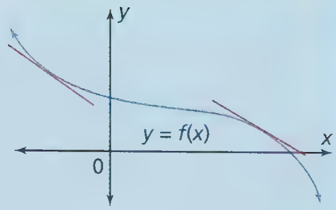
KEY IDEAS

- A function $f(x)$ is **increasing** on the open interval I $(a < x < b)$ if $f(x_1) < f(x_2)$ for all pairs of numbers, x_1 and x_2, such that $x_1 < x_2$ in I.



$f(x)$ increasing; $f'(x) > 0$

- A function $f(x)$ is **decreasing** on the open interval I ($a < x < b$) if $f(x_1) > f(x_2)$ for all pairs of numbers, x_1 and x_2, such that $x_1 < x_2$ in I.
- For a function f that is continuous and differentiable on an interval I,
  - $f(x)$ is **increasing** if $f'(x) > 0$ for all x in I
  - $f(x)$ is **decreasing** if $f'(x) < 0$ for all x in I

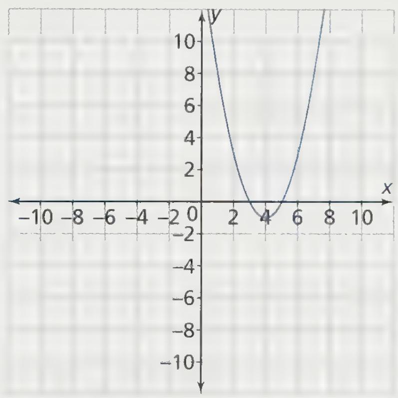


$f(x)$ decreasing; $f'(x) < 0$

4.1 Exercises

A

1. Identify the intervals on which the function increases or decreases.

(a)


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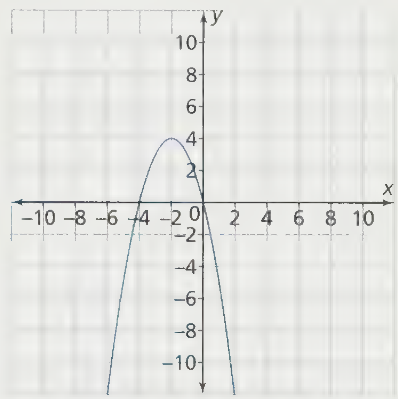

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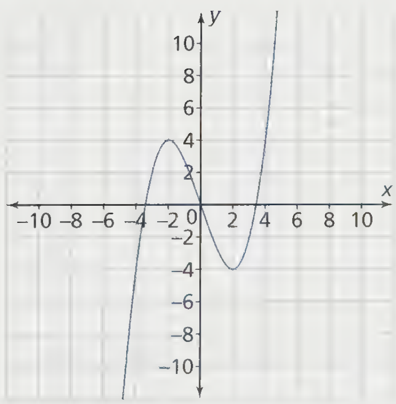

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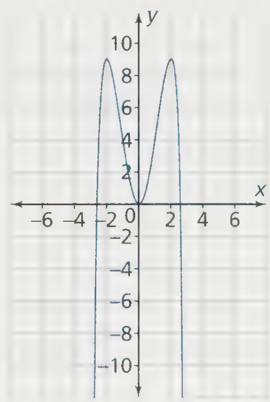
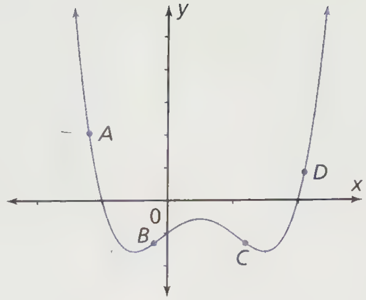


2. (a) The function $f(x)$ is an increasing function. Its derivative $f'(x)$ is defined for all $x \in \mathbf{R}$. Can any values of $f'(x)$ be negative? Explain.
   (b) The function $f(x)$ is a decreasing function. Its derivative $f'(x)$ is defined for all $x \in \mathbf{R}$. Can any values of $f'(x)$ be positive? Explain.

3. Determine the sign of $\frac{dy}{dx}$ at points A, B, C, and D.



4. Solve for x, $x \in \mathbf{R}$.

(a) $3x + 6 > 0$
(b) $-2x + 8 < 0$
(c) $(x - 5)(x + 2) > 0$
(d) $(3x + 2)(x - 4) < 0$
(e) $x^2 - 81 > 0$
(f) $x^2 - 10x + 24 > 0$
(g) $x^2 - x - 30 < 0$
(h) $8x^2 + 2x - 3 \geq 0$
(i) $2x^2 - 3x - 20 < 0$
(j) $3x^2 - 11x > 4$

5. For each function $f(x)$, determine $f'(x)$. Also determine when $f'(x) > 0$. State the intervals on which $f(x)$ is increasing.

(a) $f(x) = 2x + 10$
(b) $f(x) = -4x + 9$
(c) $f(x) = x^2 + 3$
(d) $f(x) = -2x^2 - 8$
(e) $f(x) = 4x^2 + 8x$
(f) $f(x) = -5x^2 - 20x + 3$

6. For each function $f(x)$, determine $f'(x)$. Also determine when $f'(x) < 0$. State the intervals on which $f(x)$ is decreasing.

(a) $f(x) = -3x - 12$
(b) $f(x) = 5x + 35$
(c) $f(x) = 3x^2 + 13$
(d) $f(x) = -3x^2 - 12$
(e) $f(x) = 3x^2 + 12x - 2$
(f) $f(x) = -4x^2 - 32x + 5$

B

7. Determine the intervals where each function increases and decreases.

(a) $y = 8x + 16$
(b) $y = -3x - 1$
(c) $y = 5$
(d) $y = x^2 + 4x + 1$
(e) $y = 6 - 3x^2$
(f) $y = -x^2 + 2x - 1$
(g) $y = x^3 - 12x + 15$
(h) $y = x^3 - 27x - 10$
(i) $y = -2x^3 + 6x - 2$
(j) $y = x^3 + 2$

8. Determine the intervals where each function increases and decreases.

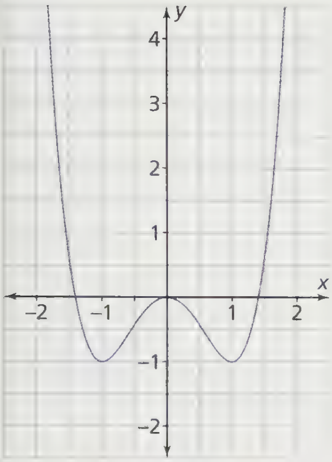
(a) $y = 2x^4 + 10$
(b) $y = -3x^4 - 12x$
(c) $y = x^4 - 2x^2 - 1$
(d) $y = 3x^4 + 4x^3 - 12x^2$
(e) $y = 2x^2 - \frac{1}{4}x^4$
(f) $y = x^4 + x^2 - 1$

9. **Knowledge and Understanding:** Determine where $g(x) = 2x^3 - 3x^2 - 12x + 15$ is increasing and where it is decreasing.

10. A plastic pop bottle holds 2 L of liquid. In an experiment, a small hole is drilled in the bottom of the bottle. The volume of liquid, V, remaining after t seconds can be modelled by $V(t) = 2 - \frac{t}{5} + \frac{t^2}{200}$, where $t \geq 0$.

(a) How long does it take for the 2 L of liquid to drain from the bottle?

(b) Verify that the volume of liquid is always decreasing until the bottle is empty.



11. **Communication:** Identify the intervals on which the function shown on the left is increasing or decreasing.

12. A slow-pitch pitcher lobs the ball toward home plate. The height of the ball in metres, h, at t seconds can be modelled by $h(t) = -4.9t^2 + 10.5t + 0.2$.

(a) When is the height of the ball increasing? decreasing?

(b) When is the velocity of the ball increasing? decreasing?

13. **Application:** The profit, P, in dollars for selling x hamburgers is modelled by $P(x) = 2.44x - \frac{x^2}{20\,000} - 5000$, where $0 \leq x \leq 35\,000$. For what quantities of hamburgers is the profit increasing? decreasing?

14. Graph f if $f'(x) < 0$ when $x < -2$ and when $x > 3$, $f'(x) > 0$ when $-2 < x < 3$, and $f(-2) = 0$ and $f(3) = 5$.

15. Graph f if $f'(x) > 0$ when $x < -3$ and when $x > 1$, $f'(x) < 0$ when $-3 < x < 1$, and $f(-3) = 4$ and $f(1) = 2$.

16. Use an example to show and verify the following: If functions f and g are increasing on an interval I, then $f + g$ must also be increasing on I.

17. The world population from 1900 to 2000 can be modelled by $P(t) = 0.0012t^3 + 0.3197t^2 + 0.2109t + 1688.951$, where P is the population in millions and t is the number of years since 1900. Did the world population ever decrease in the 20th century? Justify your answer.

18. **Thinking, Inquiry, Problem Solving:** After birth, a baby normally loses weight for a few days and then starts gaining. The table shows an infant's weight during the first two weeks.

Day	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Weight (kg)	3.14	3.03	2.95	2.80	2.77	2.76	2.79	2.84	2.93	2.95	3.01	3.14	3.32	3.49	3.68

Determine the best polynomial model for this data. Use your model to find the intervals on which the infant's weight is increasing and decreasing.

19. **Check Your Understanding:** A rock is thrown into the air from a bridge. Verify that its height decreases over the interval $1 < t < 2.2$. The height of the rock above the water, h, in metres at t seconds is modelled by $h(t) = -4.9t^2 + 9.8t + 2.1$.

C 20. Determine the intervals in which $f(x) = |x - 2| + 3$ increases and decreases.

21. For the cubic polynomial function $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$, find conditions for a, b, c, and d to ensure that, for $-\infty < x < \infty$, f is always

(a) increasing **(b)** decreasing

22. Use calculus to prove that, for any quadratic function $f(x) = ax^2 + bx + c$,

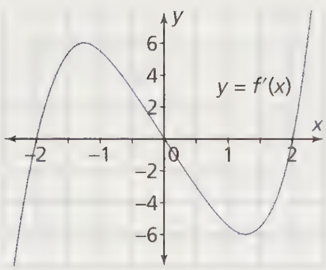
(a) if $a > 0$, then f is always decreasing when $x < -\frac{b}{2a}$ and increasing when $x > -\frac{b}{2a}$

(b) if $a < 0$, then f is always increasing when $x < -\frac{b}{2a}$ and increasing when $x > -\frac{b}{2a}$

ADDITIONAL ACHIEVEMENT CHART QUESTIONS

Knowledge and Understanding: Determine where the polynomial function $f(x) = 2x^3 - 9x^2 + 12x - 2$ is increasing and decreasing.

Application: A particle moves along a line. The particle's distance from a fixed point, s, in metres as a function of time, t, in seconds can be modelled by $s(t) = -2t^3 + 6t^2 - 3$, $t \geq 0$. Determine when the particle is moving forward.



Thinking, Inquiry, Problem Solving: Given the graph of $f'(x)$ on the left, determine where $f(x)$ is increasing and decreasing.

Communication: Suppose that you are riding on a Ferris wheel. Sketch a graph that represents your distance above the ground while you are on the Ferris wheel. When does the distance increase? When does the distance decrease?

The Chapter Problem

Trends in Post-Secondary Education

Apply what you learned in this section to answer these questions about The Chapter Problem on page 264.

CP1. Create a scatter plot using graphing technology. The independent variable is the number of years since 1980. Sketch the curve of best fit.

CP2. Using graphing technology, determine the equation of the polynomial that best models the given data.

CP3. Use the mathematical model you found to determine when the enrollment is increasing and decreasing between 1980 and 1998.

(i) $x = \frac{3}{2}$ or $x = -\frac{4}{3}$ (j) $x \doteq 3.22$ or $x \doteq -0.62$
(k) $x = -1$ or $x = 1.75$ (l) $x \doteq 1.93$ or $x \doteq -0.43$

3. (a) $x = 3, x = -5$ or $x = 7$ (b) $x = -\frac{1}{3}, x = \frac{5}{2}$ or $x = -2$
(c) $x = -1, x = 2$ or $x = 6$ (d) $x = 6, x \doteq 2.87$ or $x \doteq -0.87$
(e) $x = 6$ (f) $x = 6$ or $x = -6$
(g) $x = -0.5$ or $x = 0.5$ (h) $x \doteq 3.91$

4. (a) $x < -5$ or $x > 4$ (b) $-\frac{1}{2} < x < 3$
(c) $x < -5$ or $x > 7$ (d) $-\frac{7}{2} < x < \frac{5}{3}$
(e) $-3 < x < 1$ or $x > 6$ (f) $x < -2$ or $-\frac{3}{2} < x < \frac{1}{5}$

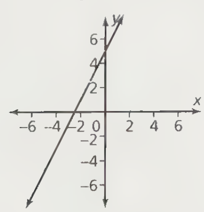
5. (a) $\frac{d}{dx}x^n = nx^{n-1}$, $n \neq 0$ where n is the coefficient; power is decreased by one; Example: $\frac{d}{dx}x^8 = 8x^7$
(b) $\frac{d}{dx}c = 0$ where c is a constant; derivative is zero; Example, $\frac{d}{dx}(10) = 0$
(c) $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$; derivative of the sum of $f(x)$ and $g(x)$ is the sum of the derivative of $f(x)$ and derivative of $g(x)$; examples will vary.
(d) $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]$; derivative of the difference of $f(x)$ and $g(x)$ is the difference of the derivative of $f(x)$ and derivative of $g(x)$; examples will vary.

6. (a) 0 (b) 4
(c) $-10x - 3$ (d) $21x^2 + 12x - 9$
(e) $20x^3 - 18x$ (f) $-6x^{-3} + 4$
(g) $-5x^{-2} - 7$ (h) $8x^3 + 36x^5 + 9$
(i) $3x^{-4} + 2x - 6$ (j) $18x + 30$

7. (a) 0 (b) 0
(c) -10 (d) $42x + 12$
(e) $60x^2 - 18$ (f) $18x^{-4}$
(g) $10x^{-3}$ (h) $24x^2 + 180x^4$
(i) $-12x^{-5} + 2$ (j) 18

8. (a) 2 (b) -1 (c) 47
(d) -114 (e) 1 (f) 426

9. (a) $x \to -\infty; y \to +\infty; x \to +\infty; y \to +\infty$
(b) $x \to -\infty; y \to +\infty; x \to +\infty; y \to -\infty$
(c) $x \to -\infty; y \to -\infty; x \to +\infty; y \to -\infty$
(d) $x \to -\infty; y \to -\infty; x \to +\infty; y \to +\infty$

10. (a)

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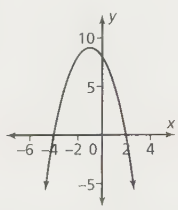
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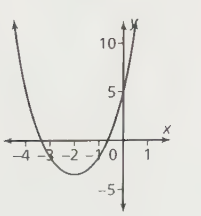
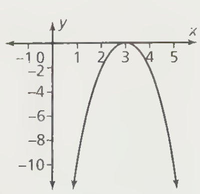
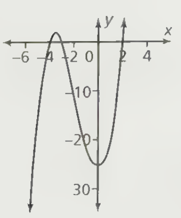
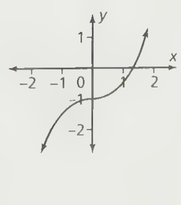

(d)

(e)

(f)


11. (a) increasing: $x > 5$; decreasing: $x < 5$; turning point: $(5, 1)$ local min.
(b) increasing: $x < -2, x > 0$; decreasing: $-2 < x < 0$; turning points: $(-2, 5)$ local max., $(0, 1)$ local min.

12. (a) increasing: $x > 1.5$; decreasing: $x < 1.5$; turning point: $(1.5, -5.75)$ local min.
(b) increasing: $x > 2.87, x < 0.46$; decreasing: $0.46 < x < 2.87$; turning points: $(2.87, -8.06)$ local min., $(0.46, -1.12)$ local min.
(c) increasing: $x > 1.5$; decreasing: $x < 1.5$; turning point: $(1.5, -1.69)$ local min.
(d) increasing: $-0.17 < x < 0.89$; decreasing: $x < -0.17, x > 0.89$; turning points: $(-0.17, -0.08)$ local min.; $(0.89, 2.15)$ local max.

13. (a) $x = 0.555$ or $x = -1.543$ (b) $x = 1.544$
(c) $x < -1.774$ or $-0.743 < x < 1.517$
(d) $x < -2.236$ or $x > 2.236$

14. $f'(0) = -2$

15. (a) $7x - y - 3 = 0$ (b) $9x - y + 15 = 0$
(c) $75x + y - 112 = 0$ (d) $5x + y - 6 = 0$

4.1 Exercises, page 273

1. (a) increasing: $x > 4$; decreasing: $x < 4$
(b) increasing: $x < -2$; decreasing: $x > -2$
(c) increasing: $x < -2; x > 2$; decreasing: $-2 < x < 2$
(d) increasing: $x < -1.8, 0 < x < 1.8$; decreasing: $-1.8 < x < 0, x > 1.8$

2. (a) no; $f'(x)$ cannot be neg.; f is an increasing function so slope of f is pos. for all x
(b) no; $f'(x)$ cannot be pos.; f is a decreasing function so slope of f is neg. for all x

3. A: neg.; B: pos.; C: neg.; D: pos.

4. (a) $x > -2$ (b) $x > 4$
(c) $x < -2$ or $x > 5$ (d) $-\frac{2}{3} < x < 4$
(e) $x < -9$ or $x > 9$ (f) $x < 4$ or $x > 6$
(g) $-5 < x < 6$ (h) $x < -0.75$ or $x > 0.5$
(i) $-2.5 < x < 4$ (j) $x < -\frac{1}{3}$ or $x > 4$

5. (a) $2; x \in \mathbf{R}; x \in \mathbf{R}$ (b) -4; no solution; no solution
(c) $2x; x > 0; x > 0$ (d) $-4x; x < 0; x < 0$
(e) $8x + 8; x > -1; x > -1$ (f) $-10x - 20; x < -2; x < -2$

6. (a) $-3; x \in \mathbf{R}; x \in \mathbf{R}$ (b) 5; no solution; no solution
(c) $6x; x < 0; x < 0$ (d) $-6x; x > 0; x > 0$
(e) $6x + 12; x < -2; x < -2$ (f) $-8x - 32; x > -4; x > -4$

7. (a) increasing: $x \in \mathbf{R}$; decreasing: none
(b) increasing: none; decreasing: $x \in \mathbf{R}$
(c) increasing: none; decreasing: none
(d) increasing: $x > -2$; decreasing: $x < -2$
(e) increasing: $x < 0$; decreasing: $x > 0$
(f) increasing: $x < 1$; decreasing: $x > 1$
(g) increasing: $x < -2, x > 2$; decreasing: $-2 < x < 2$
(h) increasing: $x < -3, x > 3$; decreasing: $-3 < x < 3$
(i) increasing: $-1 < x < 1$; decreasing: $x < -1, x > 1$
(j) increasing: $x < 0, x > 0$; decreasing: none

8. (a) increasing: $x > 0$; decreasing: $x < 0$
(b) increasing: $x < -1$; decreasing: $x > -1$
(c) increasing: $-1 < x < 0, x > 1$; decreasing: $x < -1, 0 < x < 1$
(d) increasing: $-2 < x < 0, x > 1$; decreasing: $x < -2, 0 < x < 1$
(e) increasing: $x < -2, 0 < x < 2$; decreasing: $-2 < x < 0, x > 2$
(f) increasing: $x > 0$; decreasing: $x < 0$

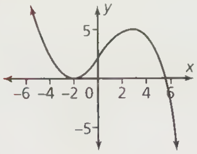
9. increasing: $x < -1, x > 2$; decreasing: $-1 < x < 2$

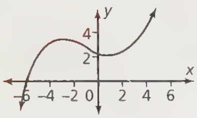
10. (a) 20 s

11. increasing: $-1 < x < 0, x > 1$; decreasing: $x < -1, 0 < x < 1$

12. (a) increasing: $t < 1.07$ s; decreasing: $t > 1.07$ s
(b) increasing: never; decreasing: $t \geq 0$

13. increasing: $0 \le x \le 24\,400$; decreasing: $24\,400 < x \le 35\,000$

14. 

15. 

17. no

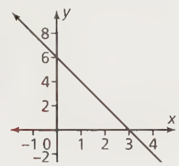
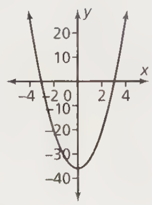
18. $f(x) = 0.0121x^2 - 0.1307x + 3.1334$; increasing $x > 5.40$; decreasing $x < 5.40$

20. decreases: $x < 2$; increases: $x > 2$

21. (a) $a > 0$; $3ac > b^2$, $d \in \mathbf{R}$
    (b) $a < 0$; $3ac > b^2$, $d \in \mathbf{R}$

4.2 Exercises, page 283

1. (a) iii. (b) iv. (c) i. (d) ii.

2. (a)  (b) 

3. (b) absolute max.: 40; absolute min.: 0

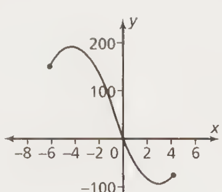
4. (a) $x = 0$ (b) absolute max.: 5; absolute min.: -11

5. (a) 0 (b) -3 (c) 4
   (d) ± 3 (e) $0, \pm\sqrt{2}$ (f) $\pm 1, \pm 2$

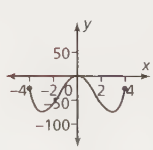
6. (a) abs. max.: 6; abs. min.: 2 (b) abs. max.: 16; abs. min.: -4
   (c) abs. max.: -1; abs. min.: -37 (d) abs. max.: 575; abs. min.: -1
   (e) abs. max.: 21; abs. min.: 1 (f) abs. max.: 23; abs. min.: -13
   (g) abs. max.: 100; abs. min.: -156 (h) abs. max.: 25; abs. min.: 0
   (i) abs. max.: 14; abs. min.: -21 (j) abs. max.: 98; abs. min.: -27

7. absolute max.: 20; absolute min.: -7

8. critical numbers: $-5, 3$;
   max.: 175; min.: -81

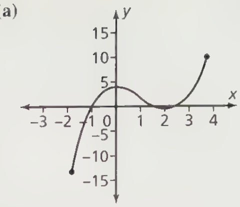


9. critical numbers: $0, \pm 3$;
   local max.: 0; local min.: -81



10. 30°C

11. no critical points: $b^2 - 3ac < 0$; Example: $x^3 + 3x^2 + 4x + 1$
    one critical point: $b^2 - 3ac = 0$; Example: $x^3 + 3x^2 + 3x + 1$
    two critical points: $b^2 - 3ac > 0$; Example: $x^3 + 3x^2 + x + 1$

12. (a) 
    (b) $\{x \mid -2 \le x \le 4, x \in \mathbf{R}\}$
    (c) increasing: $-2 \le x < 0$, $2 < x \le 4$; decreasing: $0 < x < 2$

13. absolute max.: 42; absolute min.: 10

14. $p = -2$, $q = 6$; absolute and local min. since $f(x)$ is a quadratic that opens upward, $f(0) = 6$, $f(2) = 6$

15. (a) $k < 0$ (b) $k = 0$ (c) $k > 0$

16. $a = -1$, $b = 3$, $c = 0$, $d = 0$

17. absolute max.: consider all local max. and end points—the one with the highest value is the absolute max.
    absolute min.: consider all local min. and end points—the one with the smallest value is the absolute min.

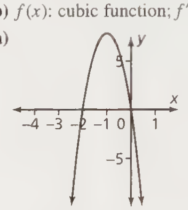
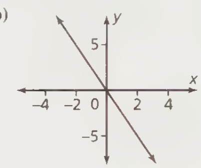
18. (a) 1

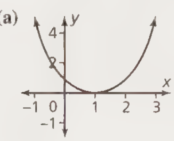
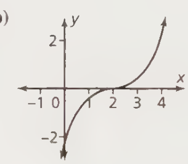
19. (a) yes, when $k \le 0$
    (b) no; either 1 or 3 but not 2

20. absolute max.: 1; absolute min.: -1

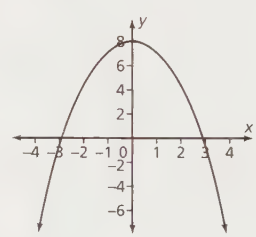
4.3 Exercises, page 292

1. (a) $f(x)$: quartic function; $f'(x)$: cubic function
   (b) $f(x)$: cubic function; $f'(x)$: quadratic function is.

2. (a)  (b) 

3. (a)  (b) 

4. $f' > 0$ over beginning of interval, then $f' < 0$ over rest of interval; f has one local max



5. $x < -2$, $-2 < x < 1$, $1 < x < 4$ and $x > 4$

6. (a) $x = -3$ at $(-3, 11)$; max.: 11; min.: none
   (b) $x = 1$ at $(1, 5)$, $x = 2$ at $(2, 4)$; max.: 5; min.: 4
   (c) $x = -3$ at $(-3, 45)$, $x = 3$ at $(3, -63)$; max.: 45; min.: -63
   (d) $x = -1$ at $(-1, 9)$, $x = 0$ at $(0, 10)$ and $x = 1$ at $(1, 9)$; max.: 10; min.: 9
   (e) $x = 0$ at $(0, 2)$, $x = 1$ at $(1, 1)$; max.: none; min.: 1
   (f) $x = -0.25$ at $(-0.25, -0.0469)$, $x = 0$ at $(0, 0)$ and $x = 1$ at $(1, -2)$; max.: 0; min.: -0.0469 and -2
   (g) $x = 0$ at $(0, 0)$, $x = -1.5$ at $(-1.5, -1.6875)$; max.: none; min.: -1.6875
   (h) $x = 0$ at $(0, 0)$, $x = 2$ at $(2, 16)$; max.: 16; min.: 0