4.3 The First Derivative Test

SETTING THE STAGE

A small rocket is launched as part of an atmospheric study. The onboard digital sensor sends back these measurements of the rocket's height at 5-s intervals.

Time (s)	0	5	10	15	20	25	30
Height (m)	1.5	754.0	1261.5	1524.0	1541.5	1314.0	841.5

What was the maximum height of the rocket?

The table shows that the maximum height is 1541.5 m at 20 s. But how do you know, without graphing, whether this maximum height is the absolute maximum height on the interval?

Identifying the critical numbers of an algebraic model will help determine the absolute maximum, or minimum, value of a function. Every local extremum occurs at a critical number of a polynomial function. But is it also true that every critical number leads to an extremum?

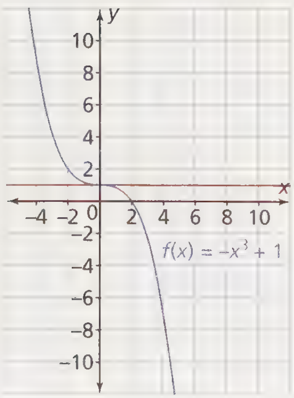
In this section, you will use the first derivative test to determine whether a critical number corresponds to a maximum or minimum value.

EXAMINING THE CONCEPT

Establishing the First Derivative Test

Example 1 **A Critical Number May Not Lead to a Maximum or Minimum**

Show that $f(x) = -x^3 + 1$ has no maximum or minimum values at the critical numbers.



Solution

To locate the critical numbers, let $f'(x) = 0$.

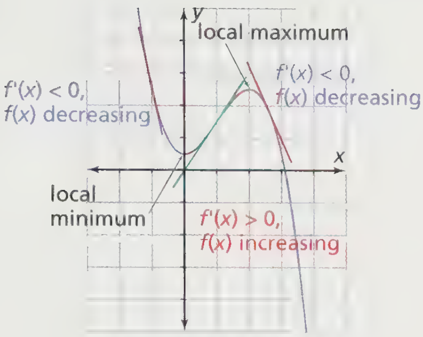
$$f'(x) = -3x^2$$
$$\therefore -3x^2 = 0$$
$$x = 0$$

The only critical point, at $x = 0$, is $(0, f(0)) = (0, -(0)^3 + 1)$, or $(0, 1)$.

Although $(0, 1)$ is a critical point, the graph of $f(x) = -x^3 + 1$ reveals that it does not correspond to an extremum. So the function does not have a maximum, or a minimum, value.

A critical number simply identifies where the slope of the tangent line is 0. So, sometimes a critical number corresponds to an extremum, and sometimes it does not.

A test is needed to determine whether a critical number corresponds to an extremum or not.



Examine the values of the derivative on both sides of a critical point to decide whether an extremum exists at that point or not.

A local minimum must exist if the graph of the function changes from falling to rising at a critical point. The sign of the derivative changes from negative to positive.

A local maximum must exist if the graph of the function changes from rising to falling at a critical point. The sign of the derivative changes from positive to negative.

The First Derivative Test for Local Extrema

Let c be a critical number of a polynomial function f that is continuous over an interval I.

- If $f'(x)$ changes from negative to positive at c, then point $(c, f(c))$ is a **local minimum point** of f.
- If $f'(x)$ changes from positive to negative at c, then point $(c, f(c))$ is a **local maximum point** of f.
- If $f'(x)$ does not change sign at c, then point $(c, f(c))$ is neither a maximum nor a minimum point.

The First Derivative Test for Absolute Extrema

Let c be a critical number of a function f that is continuous over an interval D, the domain of f.

- If $f'(x)$ is negative for all $x < c$ and $f'(x)$ is positive for all $x > c$, then $f(c)$ is the absolute minimum of f.
- If $f'(x)$ is positive for all $x < c$ and $f'(x)$ is negative for all $x > c$, then $f(c)$ is the absolute maximum of f.

Example 2 **Using the First Derivative Test to Find the Extrema of a Function**

Determine all local extrema of $g(x) = x^3 - 6x^2 + 15$.

Solution

$$g'(x) = 3x^2 - 12x$$ Determine the derivative and factor.

$$= 3x(x - 4)$$

$$3x(x - 4) = 0$$ Find the critical numbers by setting $g'(x) = 0$.

$$x = 0 \text{ or } x = 4$$

Critical numbers divide the domain, **R**, into three intervals. Use a table to analyze the sign of the derivative and the behaviour of the function in each interval.

	Intervals		
	$x < 0$	$0 < x < 4$	$x > 4$
$3x$	$-$	$+$	$+$
$x - 4$	$-$	$-$	$+$
$g'(x)$	$(-)(-) = +$	$(+)(-) = -$	$(+)(+) = +$
$g(x)$	increasing ↗	decreasing ↘	increasing ↗
	maximum at $x = 0$	minimum at $x = 4$	

The sign of the derivative changes from positive to negative at $x = 0$. So $g(x)$ has a local maximum at $(0, g(0)) = (0, 15)$. The sign of the derivative changes from negative to positive at $x = 4$. So $g(x)$ has a local minimum at $(4, g(4)) = (4, -17)$.

EXAMINING THE CONCEPT

Graphing Polynomial Functions with the First Derivative Test

Graphs are important tools for understanding the behaviour of functions and the real-world phenomena that they model. Graphing technology makes analyzing easier, but it is still necessary to understand and interpret the results. The following steps will help you to envision and graph polynomial functions quickly and accurately.

Using the First Derivative Test to Graph $y = f(x)$

1. Determine the derivative, $f'(x)$. Find all critical numbers of f.
2. Substitute each critical number into $f(x)$ to find the y-coordinate of each critical point. Plot the critical points on the coordinate plane.
3. Determine where the function increases or decreases by checking the sign of the derivative on the intervals between the critical numbers.
4. Complete the graph by ensuring that
   - the graph rises on the intervals where $f'(x) > 0$
   - the graph falls where $f'(x) < 0$
   - the graph passes through the critical points
   - the graph has horizontal tangents where $f'(x) = 0$

Example 3 Graphing a Polynomial Function Using the First Derivative Test

Graph $f(x) = x^4 - 4x^3 + 4x^2$. Classify all extrema.

Solution

Determine the derivative and factor.

$$\begin{aligned} f'(x) &= 4x^3 - 12x^2 + 8x \\ &= 4x(x^2 - 3x + 2) \\ &= 4x(x - 2)(x - 1) \end{aligned}$$

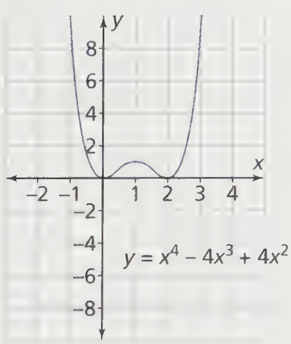
Find the critical numbers by letting $f'(x) = 0$.

$$\begin{aligned} 4x(x - 2)(x - 1) &= 0 \\ x = 0,\ x = 2,\ \text{or } x &= 1 \end{aligned}$$

Evaluate $f(x)$ at these critical numbers. So the critical points are (0, 0), (1, 1), and (2, 0).

Apply the first derivative test to determine all maximum and minimum values.

	Intervals			
	$x < 0$	$0 < x < 1$	$1 < x < 2$	$x > 2$
$4x$	$-$	$+$	$+$	$+$
$x - 2$	$-$	$-$	$-$	$+$
$x - 1$	$-$	$-$	$+$	$+$
$f'(x)$	$(-)(-)(-) = -$	$(+)(-)(-) = +$	$(+)(-)(+) = -$	$(+)(+)(+) = +$
$f(x)$	decreasing ↘	increasing ↗	decreasing ↘	increasing ↗
	minimum at $x = 0$	maximum at $x = 1$	minimum at $x = 2$	



The first derivative test indicates that (0, 0) and (2, 0) correspond to local minima of $f(x)$ and that (1, 0) corresponds to a local maximum. Both local minima are also absolute minima.

Because the degree of the polynomial is even and the leading coefficient is positive, $f(x) \to \infty$ as $x \to \infty$ and as $x \to -\infty$.

With this information, graph the function.

Example 4 Using the First Derivative Test on a Polynomial Model

Recall the problem in Setting the Stage.

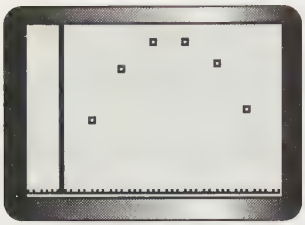
A small rocket is launched as part of an atmospheric study. The onboard digital sensor sends back these measurements of the rocket's height at 5-s intervals.

Time (s)	0	5	10	15	20	25	30
Height (m)	1.5	754.0	1261.5	1524.0	1541.5	1314.0	841.5

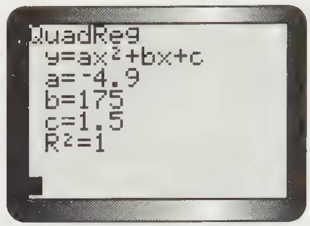
What was the maximum height of the rocket?

Solution

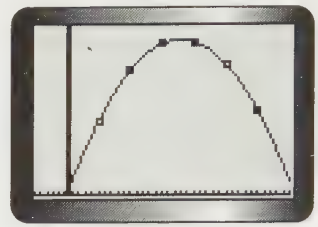
To analyze the situation represented by the discrete data, develop an algebraic model. Use graphing technology to create a scatter plot. The points appear to fit a quadratic curve. Perform a quadratic regression to determine the equation for the curve of best fit.



Scatter plot



Quadratic regression



Graph of quadratic model

The equation that models the data is $f(t) = -4.9t^2 + 175t + 1.5$, where f is the height of the rocket in metres at t seconds. Both time and height can only be positive, so the model has a restricted domain: $0 \le t \le 35.72$, since $f(35.72) \doteq 0$.

Find the derivative and the critical numbers, where $f'(t) = 0$.

$$\begin{aligned} f'(t) &= -9.8t + 175 \\ 0 &= -9.8t + 175 \\ t &\doteq 17.86 \end{aligned}$$

Apply the first derivative test.

	Intervals	
	$0 \le t < 17.86$	$17.86 < t \le 35.72$
$-9.8t + 175$	+	−
$f'(t)$	+	−
$f(t)$	increasing ↗	decreasing ↘
	maximum at $t \doteq 17.86$	

Verify this result using graphing technology.

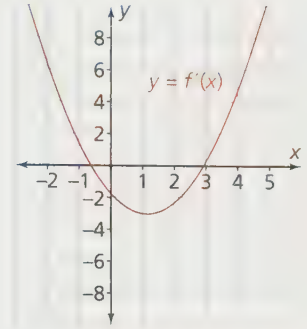
The rocket achieves its maximum height 17.86 s into the flight. Substitute $t = 17.86$ into $f(t)$ to determine the height at that time.

$$\begin{aligned} f(17.86) &= -4.9(17.86)^2 + 175(17.86) + 1.5 \\ &\doteq 1564 \end{aligned}$$

The maximum height is about 1564 m.

Example 5 Graphing a Function Given the Graph of the Derivative

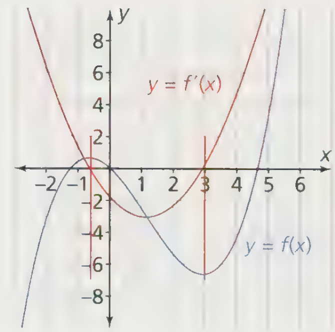
Consider the graph of $f'(x)$. Graph $f(x)$.



Solution

When the derivative, $f'(x)$, is positive, the graph of $f(x)$ is rising. When the derivative is negative, the graph is falling. In this example, the derivative changes sign at $x \doteq -0.6$ and again at $x \doteq 2.9$. The function $f(x)$ must have a local maximum and a local minimum at these numbers, respectively.

One possible graph of $f(x)$ is shown.



CHECK, CONSOLIDATE, COMMUNICATE

1. Explain how you can use the intervals of increase and decrease to decide whether a local maximum or a local minimum exists at a critical point.

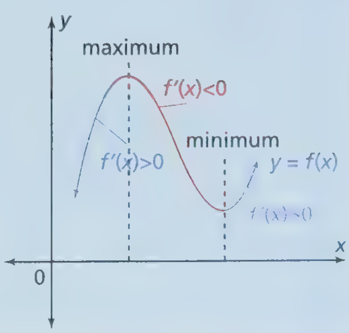
2. Can a local maximum also be an absolute maximum? Explain.

3. A function has a local maximum. What is the sign of the derivative to the left of this value? to the right?

4. A function has a local minimum.What is the sign of the derivative to the left of this value? to the right?

KEY IDEAS

- The critical numbers of a function occur where the slope of the tangent line at the critical point is 0. The first derivative test determines whether an extremum exists at a critical point.
- **The First Derivative Test for Local Extrema**
  Let c be a critical number of a polynomial function f that is continuous over an interval I.



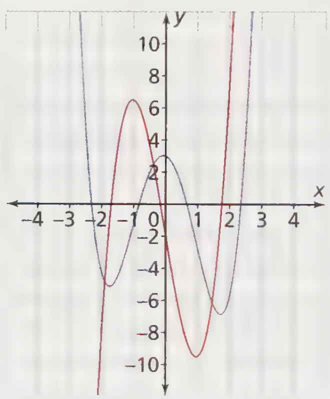
- If $f'(x)$ changes from negative to positive at c, then point $(c, f(c))$ is a local minimum of f.
- If $f'(x)$ changes from positive to negative at c, then point $(c, f(c))$ is a local maximum of f.
- If $f'(x)$ does not change sign at c, then point $(c, f(c))$ is neither a maximum nor a minimum.

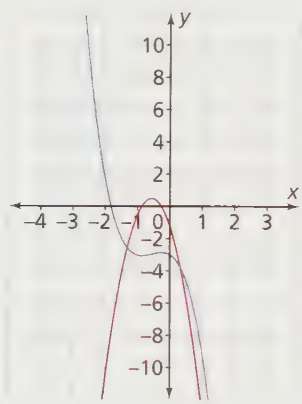
- **The First Derivative Test for Absolute Extrema**
Let c be a critical number of a function f that is continuous over an interval D, the domain of f.
  - If $f'(x)$ is negative for all $x < c$ and $f'(x)$ is positive for all $x > c$, then $f(c)$ is the absolute minimum of f.
  - If $f'(x)$ is positive for all $x < c$ and $f'(x)$ is negative for all $x > c$, then $f(c)$ is the absolute maximum of f.

4.3 Exercises

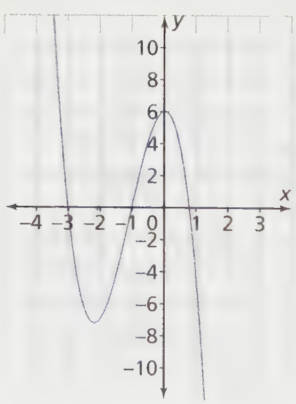
A

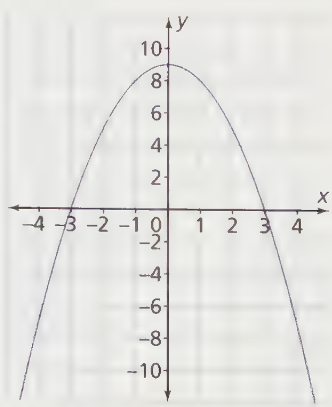
1. In each graph, which curve represents $y = f(x)$ and which represents $y = f'(x)$? Explain your choice.

(a)


(b)


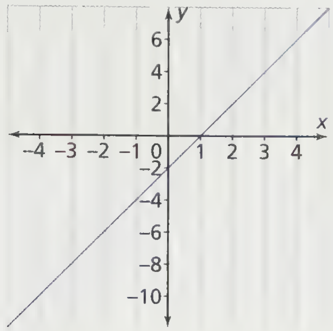
2. Each graph represents a function. Graph the derivative of each function.

(a)


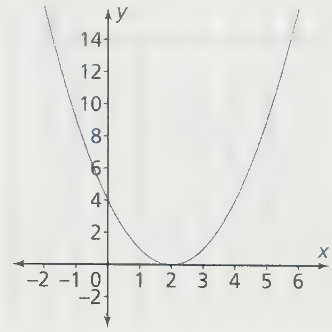
(b)


3. Each graph represents the derivative of a function. Graph a possible corresponding function.

(a)



(b)



4. As x increases over an interval, $f(x)$ increases and then decreases. Describe the behaviour of $f'(x)$. Sketch a possible graph of $f(x)$. What can you conclude about $f(x)$?

5. A polynomial function has three critical numbers: $x = -2$, $x = 1$, and $x = 4$. State the intervals on the domain created by these numbers.

B

6. For each function, find the critical numbers. Use the first derivative test to identify the local maximum and minimum values.

(a) $g(x) = 2 - 6x - x^2$ **(b)** $g(x) = 2x^3 - 9x^2 + 12x$

(c) $g(x) = x^3 - 27x - 9$ **(d)** $g(x) = x^4 - 2x^2 + 10$

(e) $g(x) = 3x^4 - 4x^3 + 2$ **(f)** $g(x) = 4x^4 - 4x^3 - 2x^2$

(g) $g(x) = x^4 + 2x^3$ **(h)** $g(x) = 12x^2 - 4x^3$

7. For each function, find the critical numbers. Determine where the function increases and decreases. Decide whether each critical point represents a maximum value, a minimum value, or neither. Use this information to graph the function.

(a) $f(x) = x^2 - 4x + 5$ **(b)** $f(x) = 10x - x^2$

(c) $f(x) = x^3 - 3x^2 + 2$ **(d)** $f(x) = x^3 - 3x + 6$

(e) $f(x) = 2x^3 - 6x^2 - 18x + 3$ **(f)** $f(x) = 2 - x^3$

(g) $f(x) = x^4 + 4x$ **(h)** $f(x) = x^4 - 6x^2 - 3$

8. **Knowledge and Understanding:** For $f(x) = x^4 - 32x + 4$, find the critical numbers, the intervals on which the function increases and decreases, and all the local extrema. Use graphing technology to verify your results.

9. Sketch a graph of the function g that is differentiable on the interval $-2 \leq x \leq 5$, decreases on $0 < x < 3$, and increases elsewhere on the domain. The absolute maximum of g is 7 and the absolute minimum is -3. The graph of g has local extrema at $(0, 4)$ and $(3, -1)$.

10. **Communication:** Graph a quartic polynomial function that has four zeros, one absolute minimum, a different local minimum, and one local maximum.

11. Find a value of k that gives $f(x) = kx^2 - 4x + 6$ an absolute maximum at $x = -2$.

12. Find a value of k that gives $f(x) = x^2 + kx + 2$ a local minimum value of 1.

13. A publishing company uses the model $P(x) = 12x - 0.0001x^2 - 10\ 000$ to estimate the profit, P, from the sale of x copies of a novel. The maximum print run for this novel is 10 000 books. How many books should be printed to maximize profit?

14. Four congruent squares are cut from the corners of a 5-cm by 8-cm piece of sheet metal. The metal is folded to form a small, open box. The volume, V, of the box is given by $V(x) = 4x^3 - 26x^2 + 40x$, where volume is measured in cubic centimetres and x is the length of each congruent square. What length x will produce a box with maximum volumè?

15. **Application:** The table shows the number of students who are absent from a large high school on certain days with the flu. Using a polynomial model, determine when the absences were at a maximum and at a minimum during this two-week period.

Day	0	3	6	9	12	14
Students Away with Flu	96	204	239	172	55	32

16. **Thinking, Inquiry, Problem Solving:** During a rocket's flight, the velocity of the rocket is recorded at 1-s intervals. Use this data to model and graph the rocket's altitude versus time.

Time (s)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/s)	0	5	10	35	65	90	110	95	55	25	0

Time (s)	11	12	13	14	15	16	17	18	19	20
Velocity (m/s)	−10	−20	−30	−40	−50	−60	−70	−80	−90	0

17. **Check Your Understanding:** Does a function always have a local maximum or minimum at every critical number? Illustrate with one or more examples.

C 18. A rectangular pen will be built with fencing that costs $25/m. The budget for the project is $1500. What are the dimensions of the pen with the largest possible area?

19. Find the point on the graph of $f(x) = 4x^3 - 3x^2 + 2x - 3$ where the slope of the tangent line represents a minimum.

20. A farmer has 500 bushels of apples to sell to a fruit store. The highest possible price is \$10 a bushel. In the past, the farmer has offered a discount of \$0.50 per bushel for every 50 bushels the store buys. At what price should the farmer sell the crop to maximize revenue?

21. Find the maximum and minimum slopes of all lines tangent to $y = 2x^3 - 8x^2 + 5x - 4$.

Maria Gaëtana Agnesi (1718–1799)

Before the 20th Century, only a few women had received credit for their mathematical contributions. Maria Agnesi was one of these women. In the 1740s she wrote a textbook that included differential and integral calculus. By 1748 she was an honorary faculty member of the University of Bologna.

ADDITIONAL ACHIEVEMENT CHART QUESTIONS

Knowledge and Understanding: For $f(x) = 3x^4 - 4x^3$,

(a) determine all critical numbers

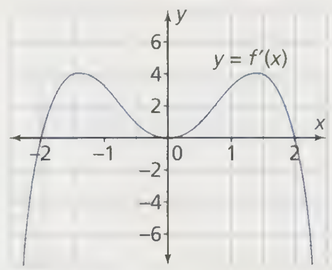
(b) determine intervals of increase and decrease

(c) determine local maximum and minimum values

(d) graph $f(x)$

Application: The position, s, of a particle moving along a line and away from a fixed point is given by $s(t) = -2t^3 + 6t^2 - 3$, $t \geq 0$. The position is measured in metres at t seconds. Determine when the particle changes direction.

Thinking, Inquiry, Problem Solving: The graph of $f'(x)$ is shown.



(a) Determine the intervals where $f(x)$ is increasing or decreasing. What are the local extrema?

(b) Graph $f(x)$.

Communication: Must f have a local maximum or minimum at $x = c$ if $y = f(x)$ is differentiable at c and $f'(c) = 0$, where c is a value in the domain of f? Explain.

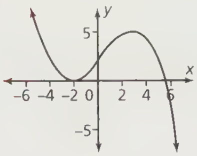
The Chapter Problem

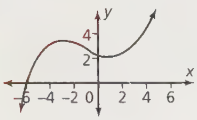
Trends in Post-Secondary Education

Apply what you learned in this section to answer this question about The Chapter Problem on page 264.

CP7. Use the first derivative test to verify the existence of the local extrema you found for question CP5 using the mathematical model.

13. increasing: $0 \le x \le 24\ 400$; decreasing: $24\ 400 < x \le 35\ 000$

14. 

15. 

17. no

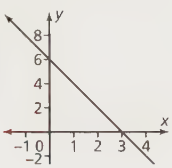
18. $f(x) = 0.0121x^2 - 0.1307x + 3.1334$; increasing $x > 5.40$; decreasing $x < 5.40$

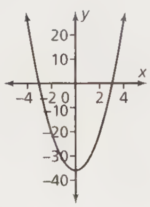
20. decreases: $x < 2$; increases: $x > 2$

21. (a) $a > 0$; $3ac > b^2$, $d \in \mathbf{R}$
(b) $a < 0$; $3ac > b^2$, $d \in \mathbf{R}$

4.2 Exercises, page 283

1. (a) iii. (b) iv. (c) i. (d) ii.

2. (a) 

(b) 

3. (b) absolute max.: 40; absolute min.: 0

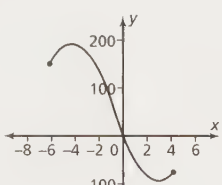
4. (a) $x = 0$ (b) absolute max.: 5; absolute min.: -11

5. (a) 0 (b) -3 (c) 4
(d) ± 3 (e) $0, \pm\sqrt{2}$ (f) $\pm 1, \pm 2$

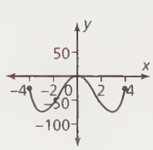
6. (a) abs. max.: 6; abs. min.: 2 (b) abs. max.: 16; abs. min.: -4
(c) abs. max.: -1; abs. min.: -37 (d) abs. max.: 575; abs. min.: -1
(e) abs. max.: 21; abs. min.: 1 (f) abs. max.: 23; abs. min.: -13
(g) abs. max.: 100; abs. min.: -156 (h) abs. max.: 25; abs. min.: 0
(i) abs. max.: 14; abs. min.: -21 (j) abs. max.: 98; abs. min.: -27

7. absolute max.: 20; absolute min.: -7

8. critical numbers: $-5, 3$;
max.: 175; min.: -81

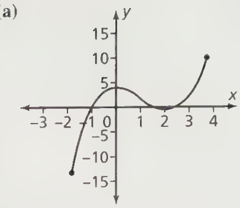


9. critical numbers: $0, \pm 3$;
local max.: 0; local min.: -81



10. 30°C

11. no critical points: $b^2 - 3ac < 0$; Example: $x^3 + 3x^2 + 4x + 1$
one critical point: $b^2 - 3ac = 0$; Example: $x^3 + 3x^2 + 3x + 1$
two critical points: $b^2 - 3ac > 0$; Example: $x^3 + 3x^2 + x + 1$

12. (a) 

(b) $\{x \mid -2 \le x \le 4, x \in \mathbf{R}\}$
(c) increasing: $-2 \le x < 0$, $2 < x \le 4$; decreasing: $0 < x < 2$

13. absolute max.: 42; absolute min.: 10

14. $p = -2$, $q = 6$; absolute and local min. since $f(x)$ is a quadratic that opens upward, $f(0) = 6$, $f(2) = 6$

15. (a) $k < 0$ (b) $k = 0$ (c) $k > 0$

16. $a = -1$, $b = 3$, $c = 0$, $d = 0$

17. absolute max.: consider all local max. and end points—the one with the highest value is the absolute max.
absolute min.: consider all local min. and end points—the one with the smallest value is the absolute min.

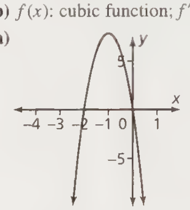
18. (a) 1

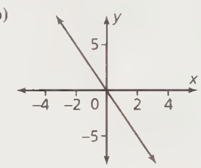
19. (a) yes, when $k \le 0$
(b) no; either 1 or 3 but not 2

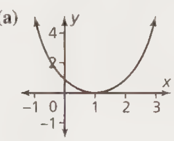
20. absolute max.: 1; absolute min.: -1

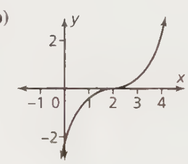
4.3 Exercises, page 292

1. (a) $f(x)$: quartic function; $f'(x)$: cubic function
(b) $f(x)$: cubic function; $f'(x)$: quadratic function is.

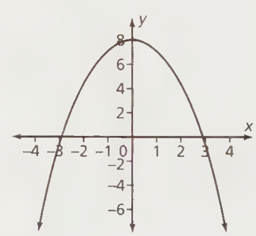
2. (a) 

(b) 

3. (a) 

(b) 

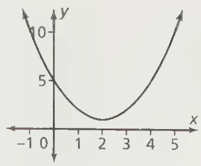
4. $f' > 0$ over beginning of interval, then $f' < 0$ over rest of interval; f has one local max



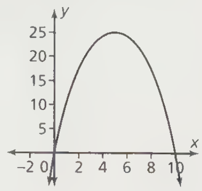
5. $x < -2$, $-2 < x < 1$, $1 < x < 4$ and $x > 4$

6. (a) $x = -3$ at $(-3, 11)$; max.: 11; min.: none
(b) $x = 1$ at $(1, 5)$, $x = 2$ at $(2, 4)$; max.: 5; min.: 4
(c) $x = -3$ at $(-3, 45)$, $x = 3$ at $(3, -63)$; max.: 45; min.: -63
(d) $x = -1$ at $(-1, 9)$, $x = 0$ at $(0, 10)$ and $x = 1$ at $(1, 9)$; max.: 10; min.: 9
(e) $x = 0$ at $(0, 2)$, $x = 1$ at $(1, 1)$; max.: none; min.: 1
(f) $x = -0.25$ at $(-0.25, -0.0469)$, $x = 0$ at $(0, 0)$ and $x = 1$ at $(1, -2)$; max.: 0; min.: -0.0469 and -2
(g) $x = 0$ at $(0, 0)$, $x = -1.5$ at $(-1.5, -1.6875)$; max.: none; min.: -1.6875
(h) $x = 0$ at $(0, 0)$, $x = 2$ at $(2, 16)$; max.: 16; min.: 0

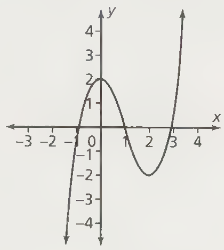
7. **(a)** $x = 2$; increasing: $x > 2$; decreasing: $x < 2$; min.: $(2, 1)$



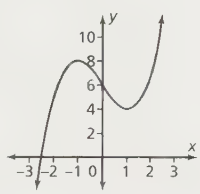
(b) $x = 5$; increasing: $x < 5$; decreasing: $x > 5$; max.: $(5, 25)$



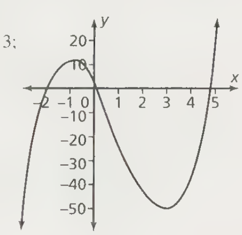
(c) $x = 0$ and $x = 2$; increasing: $x < 0, x > 2$; decreasing: $0 < x < 2$; max.: $(0, 2)$; min.: $(2, -2)$



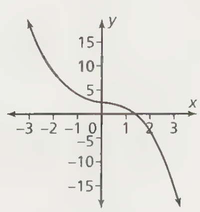
(d) $x = -1$ and $x = 1$; increasing: $x < -1$ and $x > 1$; decreasing: $-1 < x < 1$; max.: $(-1, 8)$; min.: $(1, 4)$



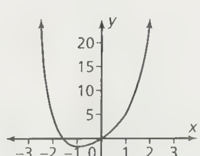
(e) $x = -1$ and $x = 3$; increasing: $x < -1, x > 3$; decreasing: $-1 < x < 3$; max.: $(-1, 13)$; min.: $(3, -51)$



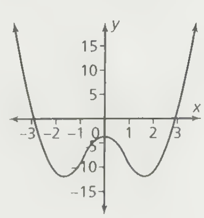
(f) $x = 0$; increasing: none; decreasing: $x \neq 0, x \in \mathbf{R}$; neither: $(0, 2)$



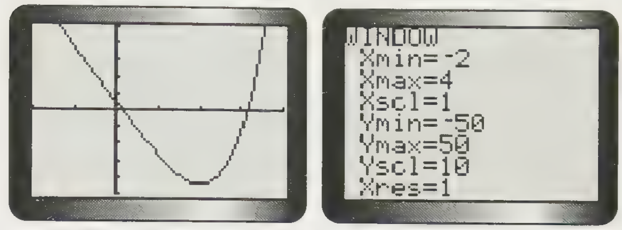
(g) $x = -1$; increasing: $x > -1$; decreasing: $x < -1$; min.: $(-1, -3)$



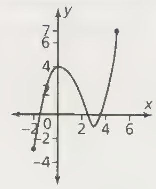
(h) $x = -\sqrt{3}, x = 0$ and $x = \sqrt{3}$; increasing: $-\sqrt{3} < x < 0, x > \sqrt{3}$; decreasing: $x < -\sqrt{3}, 0 < x < \sqrt{3}$; max.: $(0, -3)$; min.: $(-\sqrt{3}, -12)$, $(\sqrt{3}, -12)$



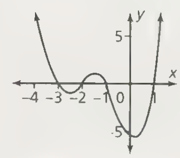
8. critical number: $x = 2$ at $(2, -44)$; increasing: $x > 2$; decreasing: $x < 2$; local min.: $(2, -44)$; local max.: none



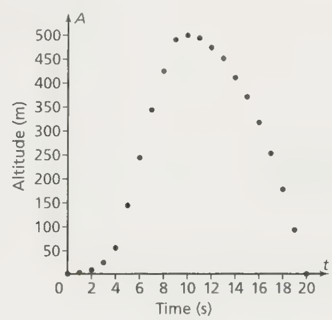
9.



10.



11. $k = 1$
12. $k = 2$ or $k = -2$
13. 10 000 books
14. $x = 1$ cm
15. max.: day 5.5; min.: day 13.6
16.



17. no; Example: $y = x^3$ — critical point is at $x = 0$, which is neither a local min. nor a local max.
18. 15 m × 15 m
19. $(0.25, -2.625)$
20. \$5/bushel
21. max. slope: none; min. slope: $-\frac{17}{3}$

4.4 Exercises, page 300

1. **(a)** $(x)(3x^2) + (1)(x^3)$
   (b) $(4x)(x^2 - 2x) + (2x^2)(2x - 2)$
   (c) $(3 - 2x^3)(4) + (-6x^2)(5 + 4x)$
   (d) $(6x + 4)(2x^2 - 9) + (3x^2 + 4x - 6)(4x)$
   (e) $(16x^3 - 16x)(4x^4 - 8x^2) + (16x^3 - 16x)(4x^4 - 8x^2)$
   (f) $(4x^3 - 2x^2 + 8x)(-2x - 5) + (12x^2 - 4x + 8)(-x^2 - 5x + 3)$
2. $F'(x) = [b(x)][c'(x)] + [b'(x)][c(x)]$
3. **(a)** $105x^6 - 120x^5 + 150x^4 - 140x^3$
   (b) $105x^6 - 120x^5 + 150x^4 - 140x^3$
   (c) same